

$$i = \sqrt{-1}$$

$$i^2 = (\sqrt{-1})^2 = \boxed{-1}$$

When simplifying, every  $i^2$  can be replaced with  $-1$  then look to continue simplifying.

When dealing with Real Numbers only:

$(x + 5)(3x + 2)$  is a Trinomial

\* When dealing with Imaginary Numbers only:

$(5 + i)(2 + 3i)$  is a Binomial

\* The product of two imaginary numbers is  
Another Imaginary Number.

Simplify each.

$$1. \quad 7i(2 - 8i)$$

$$\begin{aligned} &= 14i - 56i^2 \\ &= 14i - 56(-1) \\ &= 14i + 56 \\ &= \boxed{56 + 14i} \end{aligned}$$

$$2. \quad (1 + 6i)(4 - 3i)$$

	1	+6i	
4	4	+24i	
-3i	-12i	+18i^2 = -18(-1)	
		= +18	

$$\boxed{22 + 21i}$$

Simplify:

$$(7 + 9i)(3 - 2i)$$

	7	+9i	
3	21	+27i	
-2i	-14i	+18i^2 = +18	

$$= \boxed{39 + 13i}$$

Simplify:

$$(1 + 5i)^2$$

	1	+5i
1	1	+5i
+5i	+5i	+25i <sup>2</sup> = -25

$$= -24 + 10i$$

$(x + 5)^2$  is never just 2 terms!!!

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(x + 5)^2 = (x)^2 + 2(1)(5)x + (5)^2$$

$$= x^2 + 10x + 25$$

However.....

$$(1 + 5i)^2 = (1)^2 + 2(1)(5i) + (5i)^2$$

$$= 1 + 10i + \underbrace{25i^2}_{+25(-1)} = 1 + 10i - 25 = -24 + 10i$$

When you square a complex number you get  
another complex number.

Simplify.

$$(3 - 4i)^2$$

	3	-4i
3	9	-12i
-4i	-12i	+16i <sup>2</sup> = -16

$$-7 - 24i$$

Simplify.

$$(4 + 2i)(4 - 2i)$$

$$\begin{array}{cc|cc} & 4 & +2i & & \\ 4 & 16 & +8i & & \\ -2i & -8i & -4i^2 & = +4 & \\ \hline & & & & \end{array} = 16 + 4 = 20$$

When  $a$  and  $b$  are REAL #'s

$$(a + b)(a - b) = a^2 - b^2$$

With Imaginary Numbers:

$$(a + bi)(a - bi) = a^2 + b^2$$

$$\begin{array}{cc} (4 + 2i)(4 - 2i) = 4^2 + 2^2 = 16 + 4 = 20 \\ \swarrow \quad \searrow \\ a=4 \quad b=2 \end{array}$$

Simplify each.

$$1. (2x - 3)(2x + 3)$$

$$\begin{array}{cc|cc} & 2x & -3 & & \\ 2x & 4x^2 & -6x & & \\ +3 & +6x & -9 & & \\ \hline & & & & \end{array} = 4x^2 - 9$$

$$2. (2 - 3i)(2 + 3i)$$

$$\begin{array}{cc|cc} & 2 & -3i & & \\ 2 & 4 & -6i & & \\ +3i & +6i & -9i^2 & = +9 & \\ \hline & & & & \end{array} 4 + 9 = 13$$

Factors such as  $(a + b)$  and  $(a - b)$  are called **CONJUGATES**

Conjugate

The conjugate is where we **change the sign in the middle** of two terms like this:

$$\begin{array}{c} 3x + 1 \\ \downarrow \\ \text{Conjugate: } 3x - 1 \end{array}$$

Complex Conjugates:  $a + bi$  and  $a - bi$

$$(7 + 4i)(7 - 4i) = \begin{array}{c} 7 \quad +4i \\ \begin{array}{|c|c|} \hline 49 & +28i \\ \hline -28i & -16i^2 \\ \hline \end{array} \\ -4i \quad \quad = +16 \end{array} = 49 + 16 = 65$$

OR  $a^2 + b^2 = 7^2 + 4^2 = 49 + 16 = 65$

The product of complex conjugates is always  
a constant

Simplify each.

$$1. (9 - 5i)^2$$

$$\begin{array}{c} 9 \quad -5i \\ \begin{array}{|c|c|} \hline 81 & -45i \\ \hline -45i & +25i^2 \\ \hline \end{array} \\ -5i \quad \quad = -25 \end{array}$$

$$= 56 - 90i$$

$$2. (6 - 3i)(6 + 3i)$$

$$\begin{array}{c} a=6 \quad b=3 \\ = 6^2 + 3^2 \\ = 36 + 9 \\ = 45 \end{array}$$

$$i = \sqrt{-1} = i$$

$$i^2 = -1$$

$$i^3 = i^2 \cdot i = (-1)(i) = -i$$

$$i^4 = (i \cdot i)(i \cdot i) = (-1)(-1) = 1$$

$$i^5 = (i \cdot i)(i \cdot i) \cdot i = (-1)(-1)(i) = i$$

$$i^6 = i^3 \cdot i^3 = (-i)(-i) = i^2 = -1$$

Powers of  $i$  repeat every 4:

$i = i$	$i^5 = i$	$i^9 = i$	$i^{13} = i$
$i^2 = -1$	$i^6 = -1$	$i^{10} = -1$	$i^{14} = -1$
$i^3 = -i$	$i^7 = -i$	$i^{11} = -i$	$i^{15} = -i$
$i^4 = 1$	$i^8 = 1$	$i^{12} = 1$	$i^{16} = 1$

etc.

Simplify each.

$$1. 7i(5 + 2i)$$

$$= 35i + 14i^2$$

$$= 35i + 14(-1)$$

$$= 35i - 14$$

$$= -14 + 35i$$

$$2. (3i)(5i)(2i) =$$

$$= 30i^3$$

$$= 30(-i)$$

$$= \boxed{-30i}$$

$$3. (-2i)(10i)(i)(4i) =$$

$$= -80 \cdot i^4$$

$$= -80(1)$$

$$= \boxed{-80}$$

You can now finish Hwk #16

Sec 5-6

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Problems 37, 40, 50-52, 57, 62, 65