Sec 14-1: Trigonometric Identities

A trigonometric identity is an equation that is true for all values of x that are in the domain of the functions.

An equation in which both sides are

- ALWAYS equal
- The same
- Identical

Simplifying Trigonometric Expressions:

A trigonometric expression is an expression that contains trigonometric functions. Like all mathematical expressions, trigonometric expressions do not contain an equal sign (=).

Tools to use when simplifying Trigonometric Expressions:

Reciprocal identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

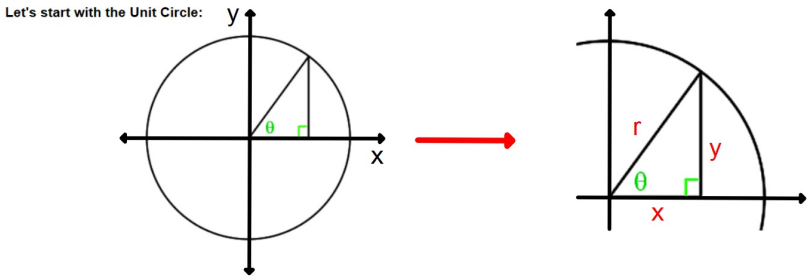
$$\cot\theta = \frac{1}{\tan\theta}$$

Tangent and cotangent identities

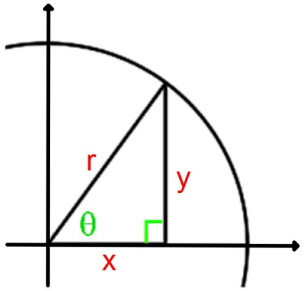
$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

The Pythagorean Identity:

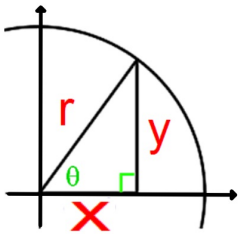
Let's start with the Unit Circle:



Using the Pythagorean Theorem we have:



$x^2 + y^2 = r^2$



$x^2 + y^2 = r^2$

Because: $x = \cos\theta$
$y = \sin\theta$
$r = 1$

substituting for x, y, and r we get:

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

this is usually written as:

$\cos^2\theta + \sin^2\theta = 1$

Pythagorean Identities:

$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$

rearranging this Pythagorean Identity leads to the following:

$Sin^2\theta = 1 - Cos^2\theta$ and $Cos^2\theta = 1 - Sin^2\theta$

Pythagorean identities

The Original Pythagorean Identity: $\cos^2\theta + \sin^2\theta = 1$

this original Pythagorean Identity can be turned into two other ones:

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \rightarrow \boxed{1 + \tan^2\theta = \sec^2\theta}$$

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \rightarrow \boxed{1 + \cot^2\theta = \csc^2\theta}$$

When you are simplifying a trigonometric expression you need to:

1. Know the rules.
2. Follow the rules.
3. Recognize that you can only multiply by 1. usually in a form similar to: sinx/sinx which is quite often used to get common denom.
4. Recognize that you can only add 0.

The rules come from definitions or identites that we have already proven.

Strategies for Simplifying Expressions

1) Change the expression into sines and cosines.
2) Look to use known formulas for purposes of substitution.
3) If there are fractions, gain a common denominator.
4) Use algebraic manipulations, like factoring, distributing, ...
5) If a strategy or substitution proves not to help, try something different.

Trigonometric Tools:

Basic Identities:

$$\text{Tan}\theta = \frac{\text{Sin}\theta}{\text{Cos}\theta}$$

$$\text{Cot}\theta = \frac{1}{\text{Tan}\theta} = \frac{\text{Cos}\theta}{\text{Sin}\theta}$$

$$\text{Csc} = \frac{1}{\text{Sin}\theta}$$

$$\text{Sec} = \frac{1}{\text{Cos}\theta}$$

Pythagorean Identities:

$$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$$

$$\text{Sin}^2\theta = 1 - \text{Cos}^2\theta$$

$$\text{Cos}^2\theta = 1 - \text{Sin}^2\theta$$

$$\text{Tan}^2\theta + 1 = \text{Sec}^2\theta$$

$$1 + \text{Cot}^2\theta = \text{Csc}^2\theta$$

Simplify each trig expression:

1. $\sin x \cot x$

$$\frac{\sin}{1} \cdot \frac{\cos}{\sin}$$

$$\cos x$$

2. $\dfrac{\sec x}{\csc x} = \dfrac{\frac{1}{\cos}}{\frac{1}{\sin}}$

$$= \frac{1}{\cos} \cdot \frac{\sin}{1}$$

$$= \frac{\sin}{\cos} = \tan x$$

Simplify each trig expression:

3. $\cos x \csc x$

$$= \frac{\cos}{1} \cdot \frac{1}{\sin}$$

$$= \frac{\cos}{\sin}$$

$$= \cot x$$

4. $\dfrac{\cos x \sec x}{\tan x} = \dfrac{\cos \cdot \frac{1}{\cos}}{\frac{\sin}{\cos}}$

$$= \frac{1}{\frac{\sin}{\cos}}$$

$$= \frac{\cos}{\sin}$$

$$= \cot x$$

Simplify each trig expression:

5. $\dfrac{\tan^2 x + 1}{1 + \cot^2 x} = \dfrac{\sec^2 x}{\csc^2 x}$

$$= \frac{\frac{1}{\cos^2}}{\frac{1}{\sin^2}}$$

$$= \frac{1}{\cos^2} \cdot \frac{\sin^2}{1}$$

$$= \frac{\sin^2}{\cos^2}$$

$$= \tan^2 x$$

6. $\dfrac{1 - \cos^2 x}{\sin^2 x}$

$$= \frac{\sin^2}{\sin^2}$$

$$= 1$$

Simplify each trig expression:

7. $(\tan x + \cot x)(\sin x \cdot \cos x)$

$$\left(\frac{\sin}{\cos} + \frac{\cos}{\sin}\right)(\sin \cdot \cos)$$

$$\sin^2 + \cos^2 = 1$$

8.

$$\frac{\sin^2 x}{\cos x \cdot \tan x} = \frac{\sin^2}{\cos \cdot \frac{\sin}{\cos}}$$

$$= \frac{\sin^2}{\sin}$$

$$= \boxed{\sin x}$$

You can now finish Hwk #24: Sec 14-1

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Problems: 18, 20-23, 28, 30, 32-34

No work = No credit