

Simplify each trigonometric expression into a single trigonometric function or a constant.

1. $\sec x \cot x - \cot x \cos x$

2. $\frac{\sin x(1 + \sin x)}{1 - \cos^2 x} - 1$

3. $\frac{\tan x - \tan x \sin^2 x}{\sin x \cos x}$

4. $\frac{\sin x}{1 - \cos x} + \frac{1 - \cos x}{\sin x}$

Simplify each trigonometric expression into a single trigonometric function or a constant.

1. $\sec x \cot x - \cot x \cos x$

$$= \frac{1}{\cos} \cdot \frac{\cos}{\sin} - \frac{\cos}{\sin} \cdot \cos$$

$$= \frac{1}{\sin} - \frac{\cos^2}{\sin}$$

$$= \frac{1 - \cos^2}{\sin}$$

$$= \frac{\sin^2}{\sin}$$

$$= \boxed{\sin x}$$

2. $\frac{\sin x(1 + \sin x)}{1 - \cos^2 x} - 1$

$$= \frac{\sin x + \sin^2 x}{\sin^2 x} - 1$$

$$= \frac{\sin x + \sin^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}$$

$$= \frac{\sin x + \sin^2 x - \sin^2 x}{\sin^2 x}$$

$$= \frac{\sin x}{\sin^2 x}$$

$$= \frac{1}{\sin x}$$

$$= \boxed{\csc x}$$

3. $\frac{\tan x - \tan x \sin^2 x}{\sin x \cos x}$

$$= \frac{\tan(1 - \sin^2)}{\sin \cdot \cos}$$

$$= \frac{\tan \cdot \cos^2}{\sin \cdot \cos}$$

$$= \frac{\tan \cdot \cos}{\sin}$$

$$= \frac{\frac{\sin}{\cos} \cdot \cos}{\sin}$$

$$= \frac{\sin}{\sin} = \boxed{1}$$

4. $\frac{\sin x}{1 - \cos x} + \frac{1 - \cos x}{\sin x}$

$$= \frac{\sin}{\sin} \cdot \frac{\sin}{1 - \cos} + \frac{1 - \cos}{\sin} \cdot \frac{1 - \cos}{1 - \cos}$$

$$= \frac{\sin^2 + 1 - 2\cos + \cos^2}{\sin(1 - \cos)}$$

$$= \frac{1 - 2\cos + \cos^2 + \sin^2}{\sin(1 - \cos)}$$

$$= \frac{1 - 2\cos + 1}{\sin(1 - \cos)}$$

$$= \frac{2 - 2\cos}{\sin(1 - \cos)}$$

$$= \frac{2(1 - \cos)}{\sin(1 - \cos)} = \frac{2}{\sin} = \boxed{2\csc x}$$