Sec 14-1: Trigonometric Identities

A trigonometric identity is an equation that is true for all values of x that are in the domain of the functions.

An equation in which both sides are The Same (Identical)

Tools to use when simplifying Trigonometric Expressions:

Reciprocal identities

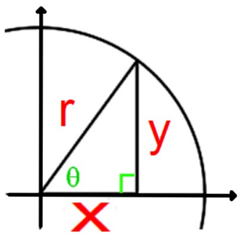
$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

Tangent and cotangent identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

The Pythagorean Identity



$x^2 + y^2 = r^2$

$x = \cos\theta$
$y = \sin\theta$
$r = 1$

substituting for x, y, and r we get:

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

this is usually written as:

$\boxed{\cos^2\theta + \sin^2\theta = 1}$

$$\cos^2\theta + \sin^2\theta = 1$$

By rearranging this Pythagorean Identity we can create the following two identities:

$$\sin^2\theta = 1 - \cos^2\theta$$

and

$$\cos^2\theta = 1 - \sin^2\theta$$

Pythagorean identities

The Original Pythagorean Identity: $\cos^2\theta + \sin^2\theta = 1$

this original Pythagorean Identity can be turned into two more identities:

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \qquad \boxed{1 + \tan^2\theta = \sec^2\theta}$$

and

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \qquad \boxed{1 + \cot^2\theta = \csc^2\theta}$$

Simplify each trig expression:

sinx cotx

$$\frac{\cancel{\sin x}}{1} \cdot \frac{\cos x}{\cancel{\sin x}} = \boxed{\cos x}$$

When you are simplifying a trigonometric expression you need to:

1. Know the rules.
2. Follow the rules.
3. Recognize that you can only multiply by ___1___.
4. Recognize that you can only add ___0___.

The rules come from definitions or identites that we have already proven.

Strategies for Simplifying Expressions

1) Change the expression into sines and cosines.
2) Look to use known formulas for purposes of substitution.
3) If there are fractions, gain a common denominator.
4) Use algebraic manipulations, like factoring, distributing, ...
5) If a strategy or substitution proves not to help, try something different.

Trigonometric Tools:

Basic Identities:

$$\text{Tan}\theta = \frac{\text{Sin}\theta}{\text{Cos}\theta}$$

$$\text{Cot}\theta = \frac{1}{\text{Tan}\theta} = \frac{\text{Cos}\theta}{\text{Sin}\theta}$$

$$\text{Csc} = \frac{1}{\text{Sin}\theta}$$

$$\text{Sec} = \frac{1}{\text{Cos}\theta}$$

Pythagorean Identities:

$$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$$

$$\text{Sin}^2\theta = 1 - \text{Cos}^2\theta$$

$$\text{Cos}^2\theta = 1 - \text{Sin}^2\theta$$

$$\text{Tan}^2\theta + 1 = \text{Sec}^2\theta$$

$$1 + \text{Cot}^2\theta = \text{Csc}^2\theta$$

$$\frac{\text{secx}}{\text{cscx}}$$

$$= \frac{\frac{1}{\cos}}{\frac{1}{\sin}} \cdot \frac{\sin}{\sin} = \frac{\frac{\sin}{\cos}}{1} = \frac{\sin}{\cos}$$

$$\tan x$$

$$= \frac{1}{\cos} \cdot \frac{\sin}{1} = \frac{\sin}{\cos} = \boxed{\text{Tan}\, x}$$

Simplify each trig expression:

sinx secx

$$= \sin \cdot \frac{1}{\cos}$$

$$= \frac{\sin}{\cos}$$

$$= \boxed{\text{Tan}x}$$

$$\frac{\text{cosx secx}}{\text{tanx}} = \frac{\cos \cdot \frac{1}{\cos}}{\frac{\sin}{\cos}} = \frac{1}{\frac{\sin}{\cos}}$$

$$= \frac{\cos}{\sin}$$

$$= \boxed{\text{COT}x}$$

Simplify each trig expression:

$$\frac{\tan^2 x + 1}{1 + \cot^2 x} = \frac{\sec^2}{\csc^2}$$

$$= \frac{\frac{1}{\cos^2}}{\frac{1}{\sin^2}}$$

$$= \frac{1}{\cos^2} \cdot \frac{\sin^2}{1} = \frac{\sin^2}{\cos^2}$$

$$= \boxed{\tan^2 x}$$

Simplify each trig expression:

$(\text{Tan}x + \text{Cot}x)(\text{Sin}x \cdot \text{Cos}x)$

$$\left(\frac{\sin}{\cos} + \frac{\cos}{\sin}\right)(\sin \cdot \cos)$$

$$\sin^2 + \cos^2$$

$$= \boxed{1}$$

$(\text{Tan}x + \text{Cot}x)(\text{Sin}x \cdot \text{Cos}x)$

$$\left(\frac{\sin}{\cos} \cdot \frac{\sin}{\sin} + \frac{\cos}{\cos} \cdot \frac{\cos}{\sin}\right)(\sin \cos)$$

$$= \left(\frac{\sin^2 + \cos^2}{\cos \cdot \sin}\right)(\sin \cdot \cos)$$

$$= \left(\frac{1}{\cancel{\cos \cdot \sin}}\right)(\cancel{\sin \cdot \cos})$$

$$= 1$$

$$\frac{\text{Cot}\theta}{\text{Csc}\theta - \text{Sin}\theta}$$

$$\frac{\frac{\cos}{\sin}}{\frac{1}{\sin} - \frac{\sin\theta}{1} \cdot \frac{\sin}{\sin}} = \frac{\frac{\cos}{\sin}}{\frac{1 - \sin^2}{\sin}} = \frac{\frac{\cos}{\sin}}{\frac{\cos^2}{\sin}}$$

$$= \frac{\cos}{\sin} \cdot \frac{\sin}{\cos^2}$$

$$= \frac{1}{\cos} = \sec x$$

You can now finish Hwk #24: Sec 14-1

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Problems: 18, 20-23, 28, 30, 32-34

No work = No credit