

The results of rolling 5 dice to simulate guessing at 5 T/F questions is shown below:

	1st hr	2nd hr	3rd hr	5th hr	Total
# times pass	129		151	53	333
# trials	240		300	90	630

Experimental probability of passing =  $\frac{333}{630} = 52.8\%$

Theoretical Probability that pass:

Probability that guess correctly on a T/F question = 0.5

Prob Guess 3 correct =  $(0.5)(0.5)(0.5)(0.5)(0.5) = 3.125\%$

correct
correct
correct
incorrect
incorrect

How many ways can you get 3 out of 5 questions correct?

Order isn't important:  ${}_5C_3 = 10$

$3.125\% \cdot 10 = 31.25\%$

Prob Guess 4 correct =  $(0.5)(0.5)(0.5)(0.5)(0.5) = 3.125\%$

correct
correct
correct
correct
incorrect

How many ways can you get 4 out of 5 questions correct?

Order isn't important:  ${}_5C_4 = 5$

$3.125\% \cdot 5 = 15.625\%$

Prob Guess 5 correct =  $(0.5)(0.5)(0.5)(0.5)(0.5) = 3.125\%$

correct
correct
correct
correct
correct

How many ways can you get 5 out of 5 questions correct?

Order isn't important:  ${}_5C_5 = 1$

$3.125\% \cdot 1 = 3.125\%$

Probability that you guess at all 5 questions  
on a T/F quiz and pass are:

3 correct OR 4 correct OR 5 correct =

3 correct = 31.25%  
+ 4 correct = 15.625%  
+ 5 correct = 3.125%

50% chance

What if there were 10 questions?

Passing would be getting:

6 correct = 20.51%

7 correct = 11.72%

8 correct = 4.39%

9 correct = 0.98%

or

10 correct = 0.10%

37.7% chance you pass by guessing

What if there were 5 multiple choice questions, with A,  
B, C, or D as choices?

3 correct = 8.79% chance

4 correct = 1.46% chance

5 correct = 0.10% chance

10.35% chance you will pass by guessing

What if there were 10 multiple choice questions, with  
A, B, C, or D as choices?

6 correct = 1.62% chance

7 correct = 0.31% chance

8 correct = 0.04% chance

9 correct = 0.003% chance

10 correct = 0.000095% chance

1.97% chance you will pass by guessing

The previous problems are examples of **Binomial Probability**.

### Binomial Probability:

- Repeated independent trials
- Only two possible outcomes: **Success** & **Failure**.
- $P(\text{Success})=s$  and  $P(\text{Failure})=f \rightarrow s + f = 1$

Probability of  $x$  successes in  $n$  trials:

$${}_nC_x \cdot s^x \cdot f^{(n-x)}$$

Probability that you guess at all 5 questions on a T/F quiz and pass are:

3 correct OR 4 correct OR 5 correct =

$$\begin{aligned} & 3 \text{ correct} = {}_5C_3 (.5)^3 (.5)^2 = 31.25\% \\ & + 4 \text{ correct} = {}_5C_4 (.5)^4 (.5)^1 = 15.625\% \\ & + 5 \text{ correct} = {}_5C_5 (.5)^5 = 3.125\% \\ & \qquad \qquad \qquad \underline{\qquad \qquad \qquad} \\ & \qquad \qquad \qquad 50\% \text{ chance} \end{aligned}$$

What if there were 5 multiple choice questions, with A, B, C, or D as choices?

$$P(\text{Success}) = .25 \quad P(\text{Failure}) = .75$$

*correct*                      *incorrect*

$$\begin{aligned} & 3 \text{ correct} = {}_5C_3 (.25)^3 (.75)^2 = 8.79\% \text{ chance} \\ & + \text{ or} \\ & 4 \text{ correct} = {}_5C_4 (.25)^4 (.75)^1 = 1.46\% \text{ chance} \\ & + \text{ or} \\ & 5 \text{ correct} = {}_5C_5 (.25)^5 = 0.10\% \text{ chance} \end{aligned}$$

10.35% chance you will pass by guessing

What if there were 10 multiple choice questions, with A, B, C, or D as choices?

$$P(\text{Success}) = .25 \quad P(\text{Failure}) = .75$$

$$\begin{aligned} & 6 \text{ correct} = {}_{10}C_6 (.25)^6 (.75)^4 = 1.62\% \text{ chance} \\ & \text{or} \\ & 7 \text{ correct} = {}_{10}C_7 (.25)^7 (.75)^3 = 0.31\% \text{ chance} \\ & \text{or} \\ & 8 \text{ correct} = \vdots = 0.04\% \text{ chance} \\ & \text{or} \\ & 9 \text{ correct} = \vdots = 0.003\% \text{ chance} \\ & \text{or} \\ & 10 \text{ correct} = \vdots = 0.000095\% \text{ chance} \end{aligned}$$

1.97% chance you will pass by guessing

Throughout the season a basketball player has made 70% of their free-throws. Find the probability that they will make 4 of their next 5 free-throws.

$$P(\text{Success}) = .70 \quad P(\text{Failure}) = .30$$

make                      miss

$$= {}^5C_4 (.70)^4 (.30)^1$$

$$= \boxed{36.02\%}$$

You plant 10 cucumber seeds in your garden. The seed company claims that their seeds will produce a plant 80% of the time.

Find the probability that at least 8 seeds will produce a cucumber plant.

$$P(\text{Success}) = .80 \quad P(\text{Failure}) = .20$$

seed germinates                      seed doesn't germinate

$$\begin{aligned} 8 &\rightarrow {}^{10}C_8 (.8)^8 (.2)^2 = 30.20\% \\ \text{or} &+ \\ 9 &\rightarrow {}^{10}C_9 (.8)^9 (.2)^1 = 26.84\% \\ \text{or} &+ \\ 10 &\rightarrow {}^{10}C_{10} (.8)^{10} = 10.74\% \\ &\quad \boxed{67.78\%} \end{aligned}$$

A nationwide survey found that 72% of people in the United States like pizza. Three people are selected at random. Find each probability as a percent to the nearest tenth.

$$P(\text{Success}) = .72 \quad P(\text{Failure}) = .28$$

LIKE PIZZA                      NOT PIZZA

1. What is the probability that all three like pizza?

$${}^3C_3 (.72)^3 = 37.32\%$$

2. What is the probability that only one of them likes pizza?

$${}^3C_1 (.72)^1 (.28)^2 = 16.93\%$$