Classwork from yesterday

1. $x =$ $y =$

$$y = \frac{1}{2}$$

1

30°

x $\frac{\sqrt{3}}{2}$

2. $x =$ $y =$

1

45°

$$y = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$x = \frac{\sqrt{2}}{2}$$

3. $x =$ $y =$

1

60°

y $\frac{\sqrt{3}}{2}$

$$x = \frac{1}{2}$$

Panel 1 (top left)
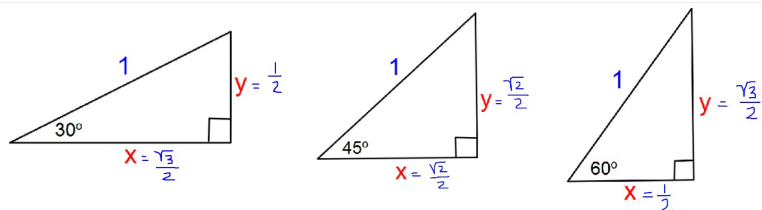

Use the above triangles to find the EXACT value of each of the below.
Rationalize denominators and simplify.

4. Sin30°

$$= \frac{\frac{1}{2}}{1} = \boxed{\frac{1}{2}}$$

5. Cos30°

$$= \frac{\frac{\sqrt{3}}{2}}{1} = \boxed{\frac{\sqrt{3}}{2}}$$

6. Tan30°

$$= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

Panel 2 (top right)

Use the above triangles to find the EXACT value of each of the below.
Rationalize denominators and simplify.

7. Sin60°

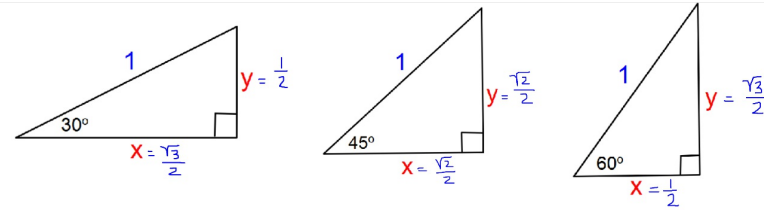$$= \frac{\frac{\sqrt{3}}{2}}{1} = \boxed{\frac{\sqrt{3}}{2}}$$

8. Cos60°

$$= \frac{\frac{1}{2}}{1} = \boxed{\frac{1}{2}}$$

9. Tan60°

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{2}{1}$$

$$= \boxed{\sqrt{3}}$$

Panel 3 (bottom left)

Use the above triangles to find the EXACT value of each of the below.
Rationalize denominators and simplify.

10. Sin45°

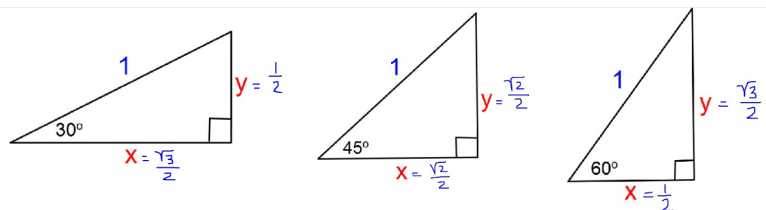$$= \frac{\frac{\sqrt{2}}{2}}{1} = \boxed{\frac{\sqrt{2}}{2}}$$

11. Cos45°

$$= \frac{\frac{\sqrt{2}}{2}}{1} = \boxed{\frac{\sqrt{2}}{2}}$$

12. Tan45°

$$= \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \boxed{1}$$

Panel 4 (bottom right)

The Unit Circle:

- Center is the origin

- Radius = 1

- Used to find the EXACT value of Sinθ, Cosθ, and Tanθ without using a calculator.

- Uses the Special Right Triangle relationships.
    This means all the angles on the unit circle are related to either 30°, 60°, or 45°.

Fill in the degrees
of this Unit Circle.
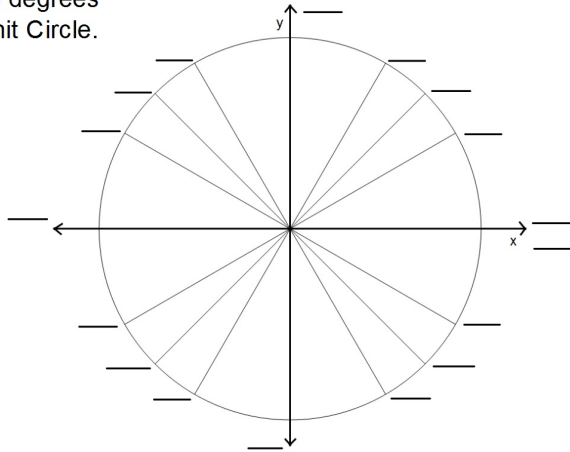
Now fill in
with radians.
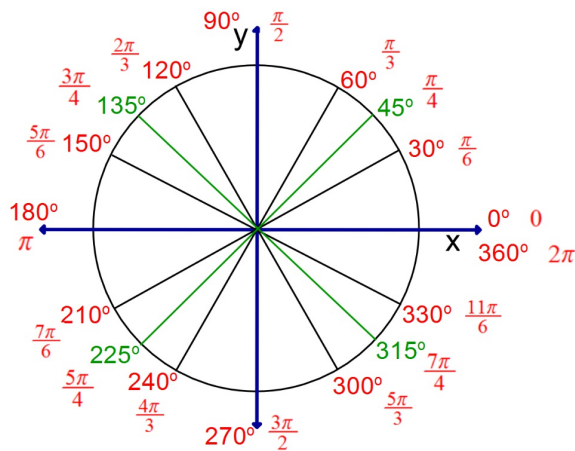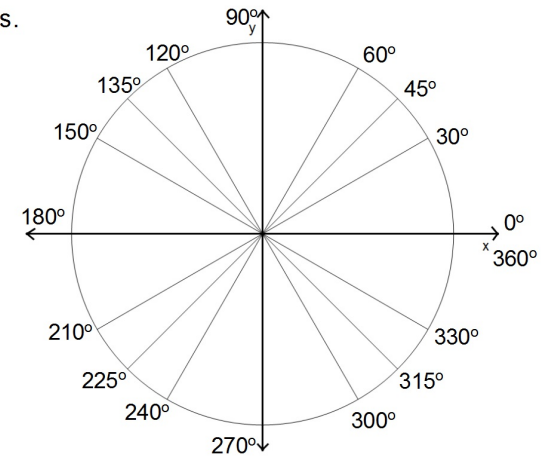
There are many ways
to remember how to fill
out the degrees and
radians on the Unit
Circle. Several ideas
were discussed in class.

But each student needs
to find a method for
completing this task
that works for them.

I will be available for
help if needed.

Coordinates of points on the Unit Circle
can be used to find the Exact Value of
Sin, Cos, and Tan of angles on the Unit Circle.

Panel 1 (top left)

(0, 1)

90° π/2
2π/3 120°
3π/4 135°
5π/6 150°
π/3 60°
π/4 45°
π/6 30°

(-1, 0) π 180°

0° 360° 2π 0 (1, 0)

7π/6 210°
225° 5π/4
240° 4π/3
3π/2 270°

330° 11π/6
315° 7π/4
300° 5π/3

(0, -1)

Fill in the coordinates on the axes.

Since this is the Unit Circle the radius is 1 and the points on the axes are just 1 unit right, 1 up, 1 left, and 1 down from the origin.

Panel 2 (top right)

The Unit Circle involves the angles in Special Right Triangles which means it probably involves the sides too!
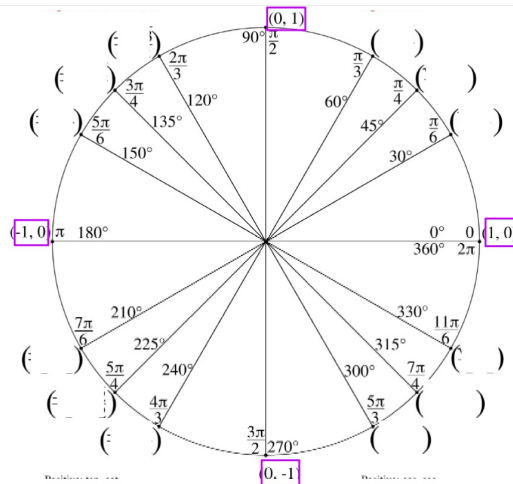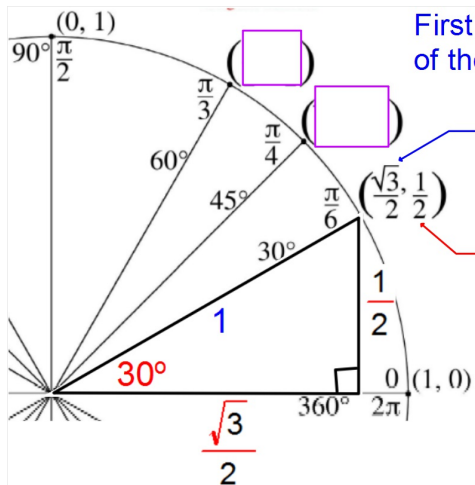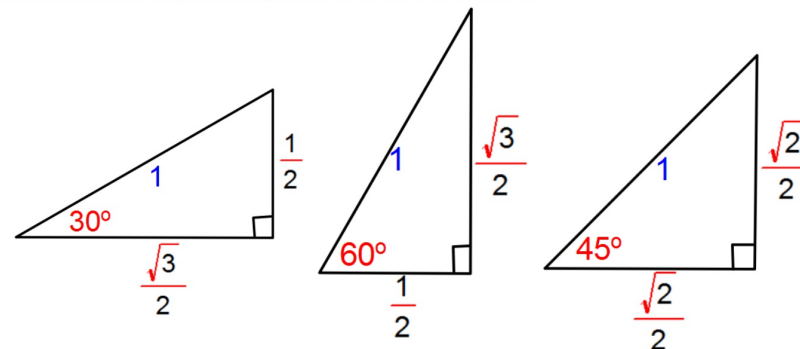

$\frac{1}{2}$ 1   30° $\frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2}$ 1   60° $\frac{1}{2}$

$\frac{\sqrt{2}}{2}$ 1   45° $\frac{\sqrt{2}}{2}$

Panel 3 (bottom left)

First Quadrant of the Unit Circle

(0, 1)
90° π/2
π/3
π/4
π/6
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
60°
45°
30°
1
$\frac{1}{2}$
30°
$\frac{\sqrt{3}}{2}$
0 (1, 0)
360° 2π

The x-coordinate is the horizontal distance from the origin which is the Long Leg of the 30-60-90 triangle.

The y-coordinate is the vertical distance from the origin which is the Short Leg of the 30-60-90 triangle.

Panel 4 (bottom right)

First Quadrant of the Unit Circle

(0, 1)
90° π/2
π/3
π/4
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
60°
45°
π/4
1
$\frac{\sqrt{2}}{2}$
45°
$\frac{\sqrt{2}}{2}$
0° 360°   0 (1, 0) 2π

The x-coordinate is the horizontal distance from the origin which is the Leg of the 45-45-90 triangle.

The y-coordinate is the vertical distance from the origin which is the Leg of the 45-45-90 triangle.

First Quadrant of the Unit Circle

The x-coordinate is the horizontal distance from the origin which is the Short Leg of the 30-60-90 triangle.

The y-coordinate is the vertical distance from the origin which is the Long Leg of the 30-60-90 triangle.

(0, 1)

$90°\ \dfrac{\pi}{2}$

$\left(\dfrac{1}{2},\ \dfrac{\sqrt3}{2}\right)$

$\dfrac{\pi}{3}$

$\dfrac{\pi}{4}$

$60°$

$\dfrac{\pi}{6}$

1

$\dfrac{\sqrt3}{2}$

$60°$

$0°$ $\quad 0$ (1, 0)

$360°\ \ 2\pi$

$\dfrac{1}{2}$

Second Quadrant of the Unit Circle

The x-coordinate is the horizontal distance from the origin which is the Short Leg of the 30-60-90 triangle.

The y-coordinate is the vertical distance from the origin which is the Long Leg of the 30-60-90 triangle.

In the Second Quadrant
the x-coordinate is negative
and the y-coordinates is positive

$\left(\dfrac{-1}{2},\ \dfrac{\sqrt3}{2}\right)$

(0, 1)

$90°\ \dfrac{\pi}{2}$

$\dfrac{2\pi}{3}$

$\dfrac{3\pi}{4}$

$120°$

$\dfrac{5\pi}{6}$

$\dfrac{\sqrt3}{2}$

1

$(-1, 0)\ \pi\ \ 180°$

$60°$

$\dfrac{1}{2}$

Second Quadrant of the Unit Circle

The x-coordinate is the horizontal distance from the origin which is the Leg of the 45-45-90 triangle.

The y-coordinate is the vertical distance from the origin which is the Leg of the 45-45-90 triangle.

In the Second Quadrant
the x-coordinate is negative
and the y-coordinates is positive

(0, 1)

$90°\ \dfrac{\pi}{2}$

$\left(\dfrac{-\sqrt2}{2},\ \dfrac{\sqrt2}{2}\right)$

$\dfrac{2\pi}{3}$

$\dfrac{3\pi}{4}$

$120°$

$135°$

$\dfrac{5\pi}{6}$

$\dfrac{\sqrt2}{2}$

1

$(-1, 0)\ \pi\ \ 180°$

$45°$

$\dfrac{\sqrt2}{2}$

Second Quadrant of the Unit Circle

The x-coordinate is the horizontal distance from the origin which is the Long Leg of the 30-60-90 triangle.

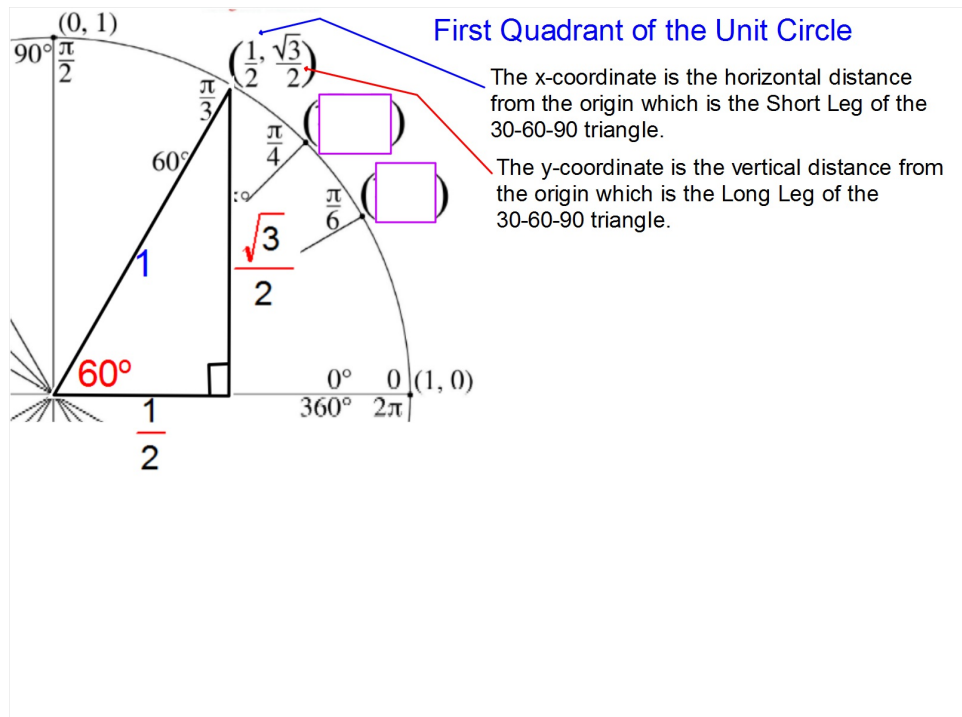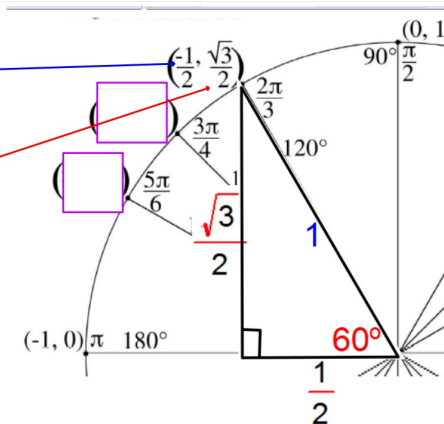The y-coordinate is the vertical distance from the origin which is the Short Leg of the 30-60-90 triangle.

In the Second Quadrant
the x-coordinate is negative
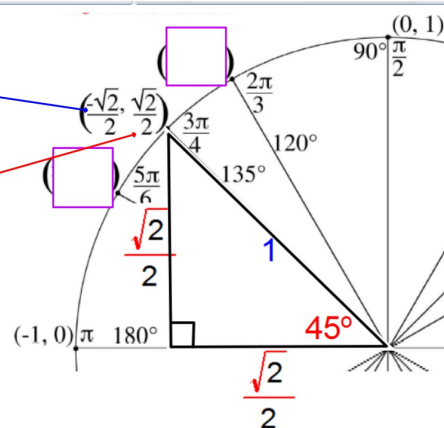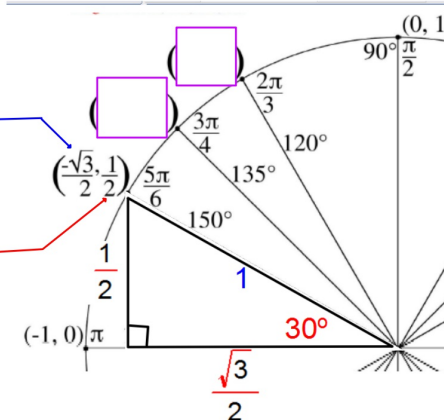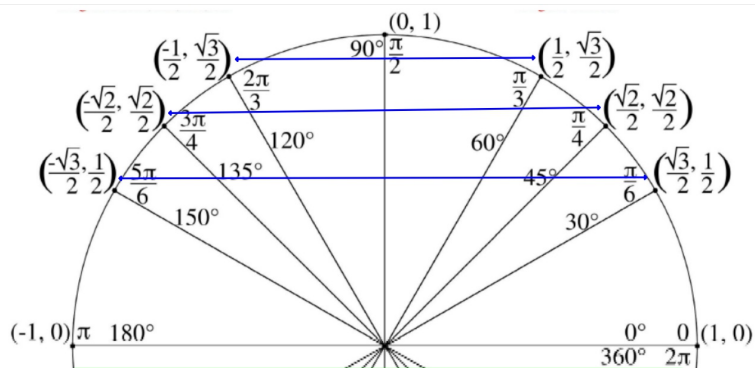and the y-coordinates is positive

(0, 1)

$90°\ \dfrac{\pi}{2}$

$\dfrac{2\pi}{3}$

$\dfrac{3\pi}{4}$

$120°$

$135°$

$\left(\dfrac{-\sqrt3}{2},\ \dfrac{1}{2}\right)$

$\dfrac{5\pi}{6}$

$150°$

$\dfrac{1}{2}$

1

$(-1, 0)\ \pi$

$30°$

$\dfrac{\sqrt3}{2}$

First Circle (1st and 2nd Quadrants)

Labeled points and angles:
- $(0, 1)$ at $90°$, $\frac{\pi}{2}$
- $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$ at $\frac{2\pi}{3}$, $120°$
- $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ at $\frac{\pi}{3}$, $60°$
- $\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ at $\frac{3\pi}{4}$, $135°$
- $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ at $\frac{\pi}{4}$, $45°$
- $\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$ at $\frac{5\pi}{6}$, $150°$
- $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ at $\frac{\pi}{6}$, $30°$
- $(-1, 0)$, π, $180°$
- $0°$, $360°$, 0, 2π, $(1, 0)$

What do you notice about the relationship amongst the coordinates in the 1st and 2nd Quadrant?

The coordinates are symmetrical about the y-axis. But in the second quadrant the x-coordinate is negative.

Second Circle (Fill in 3rd and 4th Quadrants)

Labeled points and angles:
- $(0, 1)$ at $90°$, $\frac{\pi}{2}$
- $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$ at $\frac{2\pi}{3}$, $120°$
- $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ at $\frac{\pi}{3}$, $60°$
- $\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ at $\frac{3\pi}{4}$, $135°$
- $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ at $\frac{\pi}{4}$, $45°$
- $\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$ at $\frac{5\pi}{6}$, $150°$
- $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ at $\frac{\pi}{6}$, $30°$
- $(-1, 0)$, π, $180°$
- $0°$, $360°$, 0, 2π, $(1, 0)$
- $\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$ at $\frac{7\pi}{6}$, $210°$
- $\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$ at $\frac{11\pi}{6}$, $330°$
- $\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$ at $\frac{5\pi}{4}$, $225°$
- $\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$ at $\frac{7\pi}{4}$, $315°$
- $\left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$ at $\frac{4\pi}{3}$, $240°$
- $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ at $\frac{5\pi}{3}$, $300°$
- $(0, -1)$ at $\frac{3\pi}{2}$, $270°$
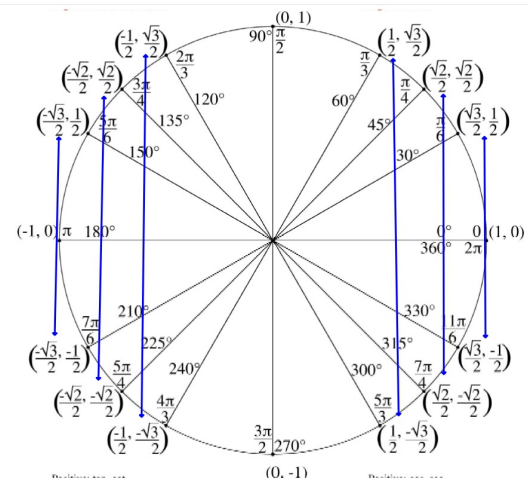
Fill in the 3rd and 4th Quadrants.

You can use 30-60-90 and 45-45-90 triangles or use the fact that the coordinates in the 3rd and 4th quadrants are reflections over the x-axis from the coordinates in the 2nd and 1st quadrants. In the 3rd quadrant x and y are both negative and in the 4th quadrant only the y-coordinate is negative.

You will have a quiz just over filling out the Unit Circle!