Use the given information to find the first term of this arithmetic sequence.

$S_{12} = -306 \qquad d = -7 \qquad n = 12$

this is actually 3 pieces of information!

You need two equations:

$-306 = \frac{12}{2}(a_1 + a_n)$

$-306 = 6(a_1 + a_n)$

$-51 = a_1 + a_n$

$a_n = a_1 - 7(12-1)$

$a_n = a_1 - 77$

$-51 = a_1 + a_1 - 77$

$26 = 2a_1$

$13 = a_1$

You can now finish Hwk #33:

Practice Sheet - Sec 11-4: Arithmetic Series

Due tomorrow

Sec 11-5: Geometric Series

A Geomteric Series is the sum of the terms of a Geometric Sequence.

Evaluate this geometric series:

6 + 12 + 24 + 48 +...+ 1536

There is nothing "obvious" about how to quickly find the sum of a geometric series like there was when finding the sum of an arithmetic series.

But, there is a formula you can use to find the sum of a finite Geometric Series.

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

S_n = the sum of the first n terms
a_1 = the first term
r = the common ratio
n = # of terms

What doesn't this formula use that you need to know for an arithmetic series? You don't need the last term!

Evaluate this first 14 terms of this series:
4 + 12 + 36 + 108 + ...

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$a_1 = 4$
$r = 3$
$n = 14$

$$S_{14} = \frac{4(1-3^{14})}{1-3}$$

$$S_{14} = 9{,}565{,}936$$

Find the sum of the first 12 terms of this geometric series:
20480 + -10240 + 5120 + -2560 + ...

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$a_1 = 20480$
$r = -.5$
$n = 12$

$$S_{12} = \frac{20480(1-(-.5)^{12})}{1-{-.5}}$$

$$S_{12} = 13{,}650$$

Evaluate this series: 6 + 12 + 24 + 48 +...+ 1536

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$a_1 = 6$
$r = 2$
$n = 9$

$$S_9 = \frac{6(1-2^9)}{1-2}$$

$$\boxed{S_9 = 3066}$$

Find the number of terms:

$a_n = a_1 \cdot r^{n-1}$
$1536 = 6(2)^{n-1}$
$256 = 2^{n-1}$
$\log_2 256 = n-1$
$\frac{\log 256}{\log 2} = n-1$
$8 = n-1$
$9 = n$

Every finite series has a sum.

What about an infinite series?

Which of the following infinite series will have a sum, if any?

1. 29+26+23+20+...

Arithmetic

NO Sum

2. 192+144+108+81,...

Geo $r = 3/4$

Has a sum

3. 1.28+ 1.60+ 2+ 2.5+ ...

Geo $r = 1.25$

Infinite Series:

The only infinite series that have a sum are

Geometric Series when $|r|<1$

Our Textbook states that if an infinite geometric series has $|r|<1$, then the series will CONVERGE, and it will have a sum.

If an infinite geometric series has $|r|>1$, then it will DIVERGE, and will not have a sum.

To find the sum of an infinite geometric series if $|r|<1$, use the following formula:

$$S = \frac{a_1}{1-r}$$

$S =$ the sum of all infinite terms

$a_1 =$ the first term

$r =$ the common ratio

How did this formula → become this formula?

Sum of a finite geometric series.

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

→

Sum of an infinite geometric series, $|r|<1$

$$S = \frac{a_1}{1-r}$$

Since r is less than one and it is being raised to a very large power (∞) it becomes so small that it essentially become zero.
Therefore, $(1-r^n)$ becomes (1-0) and $a_1(1-0) = a_1(1) = a_1$