Two types of sequences

Infinite Sequence

A sequence that
goes on forever.
It has no last term.

Finite Sequence

A sequence that
has a last term.

Some infinite sequences have a LIMIT.

When the terms of an infinite sequence get closer and closer to a fixed number L, then L is called the limit of the sequence.

If there is a limit it will be the Horizontal Asymptote of the graph of the terms of the sequence.

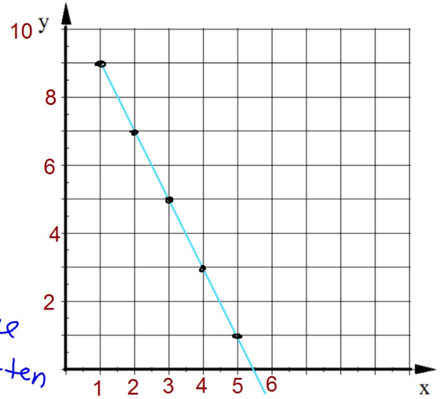
Graph the terms of this sequence to determine the limit, if there is one.

9, 7, 5, 3,1, ...

$a_n = 9 - 2(n-1)$

$= -2n + 11$

$\boxed{\text{NO LIMIT}}$, the graph will decrease for ever & not flatten out.



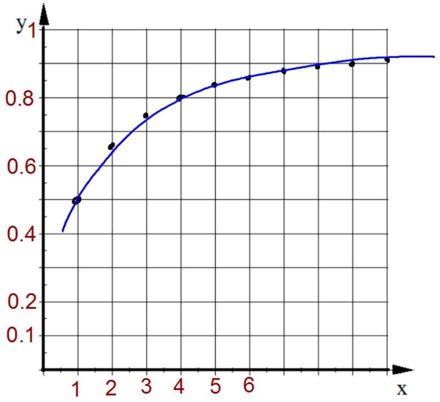
Graph the terms of this sequence to determine the limit, if there is one.

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

$\frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}, \frac{10}{11} \ldots \frac{999}{1000}$

$\boxed{\text{Limit} = 1}$

the graph flattens out an approaches 1.



The limit of an arithmetic sequence:

NO infinite arithmetic sequences have a limit.

If a sequence has a limit you can also find it by entering the Explicit Formula into Y= and substituting bigger and bigger numbers for x to see what value the sequence is approaching.

$$\overset{1}{\frac{1}{2}}, \overset{2}{\frac{2}{3}}, \overset{3}{\frac{3}{4}}, \overset{4}{\frac{4}{5}}, \ldots$$

Explicit Formula: $a_n = \frac{n}{n+1}$

If you enter this in Y=
and substitute bigger &
bigger values for n
this will get closer &
closer to 1.
therefore, Limit = 1

If the explicit formula is a Rational Function you can use this fact to help find the limit of the sequence, if there is one.

Rational Function: $\frac{\text{Polynomial}}{\text{Polynomial}}$

Reminder of what you learned last year:

Horizontal Asymptotes (HA) of Rational Functions.

- If the degree of the numerator and denominator are the same the HA is y = ratio of the leading coefficients.
- If the degree of the denominator is larger than the degree of the numerator the HA is y = 0
- If the degree of the numerator is larger than the degree of the denominator there is NO HA.

The Horizontal Asymptote of the explicit formula would be the LIMIT of the sequence.

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

Explicit Formula: $a_n = \frac{n}{n+1}$

What is the HA, if any? $Y = \frac{1}{1} = 1$

degree of numerator = degree of denominator

What is the Limit, if any? $\boxed{\text{Limit} = 1}$

Same as the HA

Do these sequences appear to have a limit? If yes, what is the limit?

A. $4, 20, 100, 500, \ldots$

$\boxed{\text{NO Limit}}$

terms keep getting bigger & bigger positive

B. $\overset{1}{\frac{3}{4}}, \overset{2}{\frac{6}{5}}, \overset{3}{\frac{9}{6}}, \overset{4}{\frac{12}{7}}, \overset{5}{\frac{15}{8}}, \ldots$

$a_n = \frac{3n}{n+3}$

HA $y = \frac{3}{1} = 3$

$\boxed{\text{Limit} = 3}$

Do these sequences appear to have a limit? If yes, what is the limit?

A. $\overset{1}{\frac{1}{2}}, \overset{2}{\frac{1}{4}}, \overset{3}{\frac{1}{8}}, \overset{4}{\frac{1}{16}}, \ldots$

$a_n = \frac{1}{2^n}$

The terms will continue to decrease but never become zero or negative

$\boxed{\text{Limit} = 0}$

B. $13, 9, 5, 1, \ldots$

ARITHMETIC

$\boxed{\text{NO LIMIT}}$

no arithmetic sequences have a limit.

Do these sequences appear to have a limit? If yes, what is the limit?

A. $486, 162, 54, 18, \ldots$

Geometric

$r = \frac{1}{3}$

The terms will continue to decrease but never become zero or negative.

$\boxed{\text{Limit} = 0}$

B. $\overset{1}{\frac{4}{10}}, \overset{2}{\frac{9}{20}}, \overset{3}{\frac{16}{30}}, \overset{4}{\frac{25}{40}}, \ldots$

$a_n = \frac{(n+1)^2}{10n}$

degree of numerator is greater than degree of the denominator.

NO HA

$\boxed{\text{NO Limit}}$

Does this sequence appear to have a limit? If yes, what is the limit?

$n = 1 \quad 2 \quad 3 \quad 4$

$$\frac{1}{10}, \frac{8}{40}, \frac{27}{90}, \frac{64}{160}, \ldots$$

$$a_n = \frac{n^3}{10n^2}$$

For this rational equation,
the degree of the numerator is greater
than the degree of the denominator.
Therefore, there is no HA.

Thus, there is NOT LIMIT.

Does this sequence appear to have a limit? If yes, what is the limit?

$n = 1 \quad 2 \quad 3 \quad 4 \quad 5$

$$\frac{50}{16}, \frac{100}{25}, \frac{150}{36}, \frac{200}{49}, \frac{250}{64}, \ldots$$

$$a_n = \frac{50n}{(n+3)^2}$$

For this rational equation,
the degree of the denominator is greater
than the degree of the numerator.
Therefore, the HA is y =0.

Thus, the LIMIT = 0

Does this sequence appear to have a limit? If yes, what is the limit?

$n = 1 \quad 2 \quad 3 \quad 4 \quad 5$

$$\frac{7}{2}, \frac{8}{4}, \frac{9}{6}, \frac{10}{8}, \frac{11}{10}, \ldots$$

$$a_n = \frac{n+6}{2n}$$

For this rational equation,
the degrees of the denominator and
numerator are equal.
Therefore, the HA is the ratio of the
leading coefficients: y = 1/2

Thus, the LIMIT = 1/2