Equations of Ellipses:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

a^2 is always the larger denominator

$c^2 = a^2 - b^2$

Equations of Hyperbolas:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

a^2 is always the denominator of the positive ratio

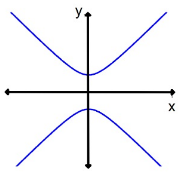
$c^2 = a^2 + b^2$

Ellipses		Hyperbolas
Distance from Center to Vertices	a	Distance from Center to Vertices
Distance from Center to Foci	c	Distance from Center to Foci
Distance from Center to Co-Vertices	b	Part of the slope of the Asymptotes
Midpoint of the Vertices, of the Co-Vertices and of the Foci. And where Axes intersect.	Center	Midpoint of the Vertices and of the Foci And where the Asymptotes intersect.
Major Axis	Segment connecting Vertices	Transverse Axis

The direction the branches of the Hyperbola open is determined by....

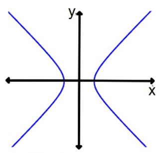
The variable in the numerator of the POSITIVE ratio.

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



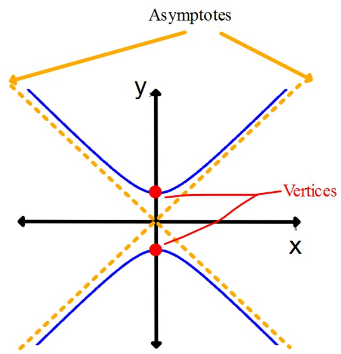
y is in the numerator of the positive ratio, therefore, the braches open in the y direction.

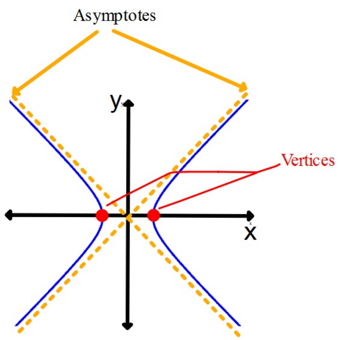
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



x is in the numerator of the positive ratio, therefore, the braches open in the x direction.

Graphs of Hyperbolas have two asymptotes.

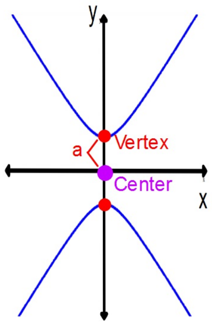




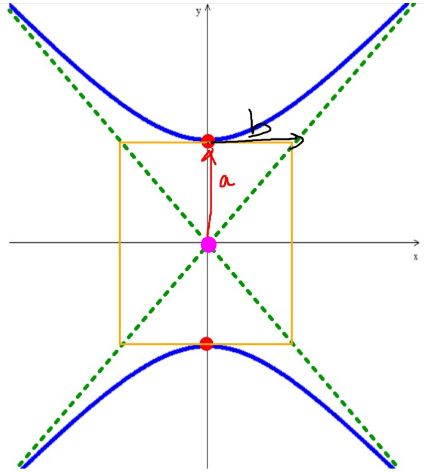
a is the distance from the center to each Vertex

What is b?

It is a value to help find the slope of the Asymptotes. See the next two screens.



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

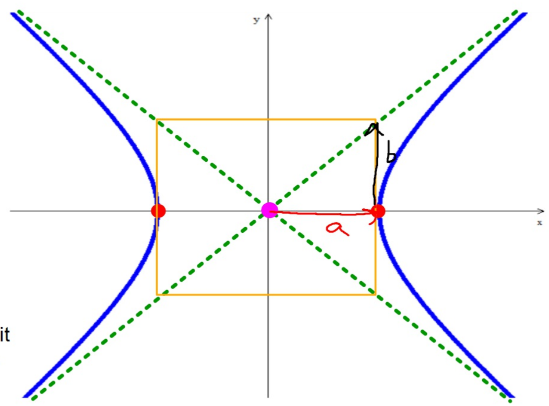


Asymptotes:

$$y = \pm \frac{a}{b} x$$

since a is measured vertically it is the rise and b must be the run.

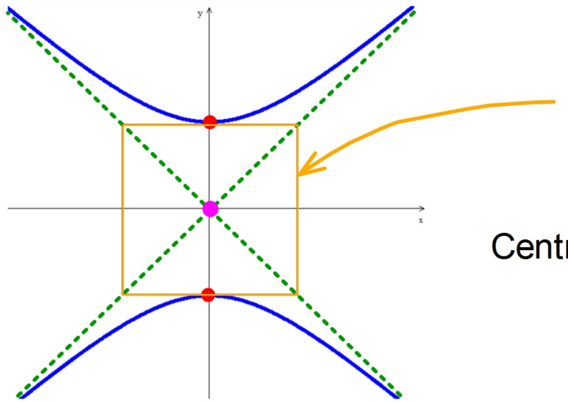
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Asymptotes:

$$y = \pm \frac{b}{a} x$$

since a is measured horizontally it is the run and b must be the rise.



Our book refers to this as the

Central Rectangle

New vocabulary for a Hyperbola:

Transverse Axis: Segment connecting the Vertices.

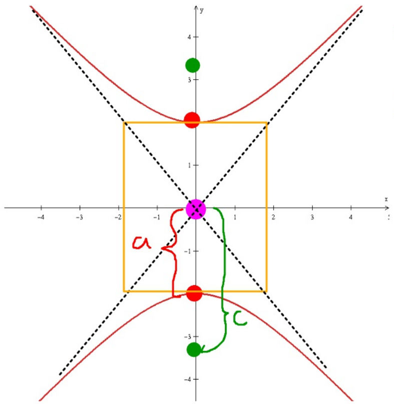
Length of the Transverse Axis = 2a

Just like in an Ellipse:

a is the distance from the center to a Vertex

c is the distance from the center to a Focus

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



Center: (0,0)

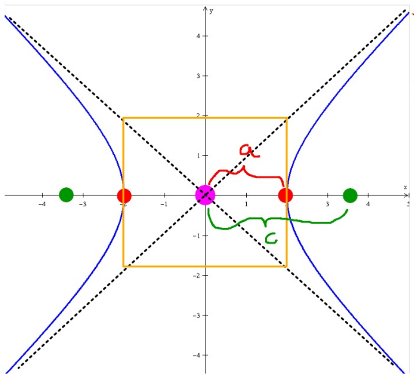
Vertices: $(0,\pm a)$

Foci: $(0,\pm c)$

Transverse Axis: Vertical
Length: 2a units

Asymptotes: $y = \pm\frac{a}{b}x$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Center: (0,0)

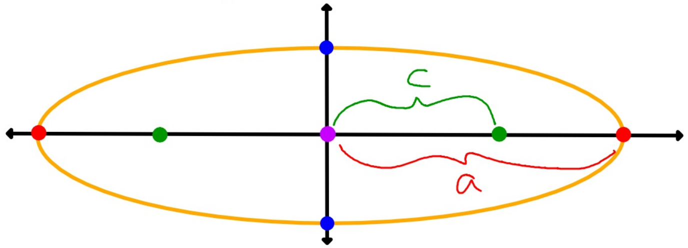
Vertices: $(\pm a,0)$

Foci: $(\pm c,0)$

Transverse Axis: Horizontal
Length: 2a units

Asymptotes: $y = \pm\frac{b}{a}x$

In an Ellipse: $c^2 = a^2 - b^2$

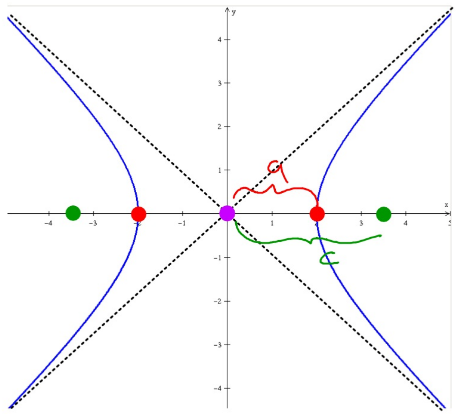


C is smaller than a because the Focus is closer to the Center than the Vertex.

Therefore, C^2 is smaller than a^2 so it makes sense to subtract b^2 from a^2.

In a Hyperbola: Relationship between a^2, b^2, and c^2:

$$c^2=a^2+b^2$$



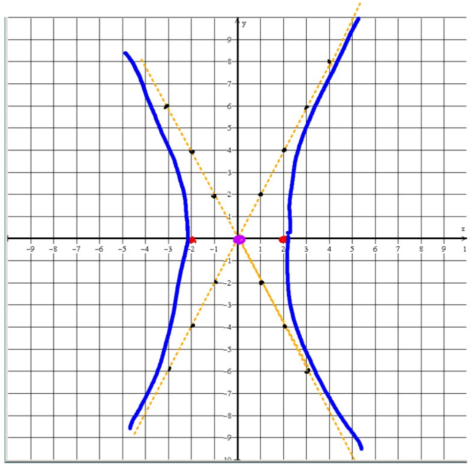
C is bigger than a because the Focus is farther away from the Center than the Vertex.

Therefore, C^2 is bigger than a^2 so it makes sense to add b^2 to a^2.

Graph this hyperbola.
Show the vertices and asymptotes.

$$\frac{x^2}{4} - \frac{y^2}{16} = 1$$

1. Center $(0,0)$
2. Vertices $(\pm 2, 0)$
3. Asymptotes $y = \pm \frac{4}{2} x$
4. Graph

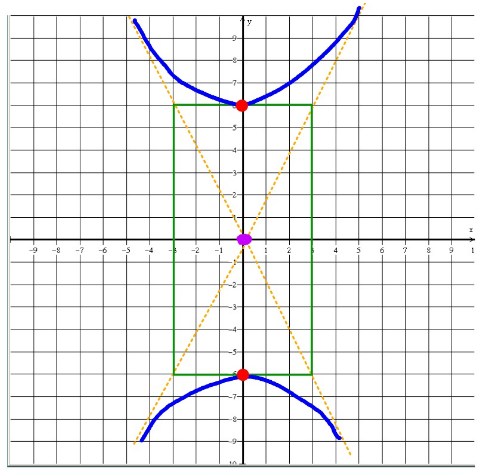


Graph this hyperbola.
Show the vertices and asymptotes.

$$\frac{y^2}{36} - \frac{x^2}{9} = 1$$

$a^2 = 36$
$a = 6$

1. Center $(0,0)$
2. Vertices $(0, \pm 6)$
3. Asymptotes $y = \pm \frac{6}{3} x$
4. Graph



You could use the Central Rectangle to find the Asymptotes by drawing horizontal lines through the vertices for the top and bottom of the rectangle then vertical lines b units left and right of the center for the sides. Once the Central Rectangle has been created the asymptotes are found by connecting opposite corners.

1. State the coordinates of the Vertices, the length of the Transverse Axis, the coordinates of the Foci, and the slopes of the asymptotes.

a. $\frac{y^2}{81} - \frac{x^2}{49} = 1$

$b^2 = 49 \quad b = 7$

$a^2 = 81$
$a = 9$

$c^2 = a^2 + b^2$
$= 81 + 49$
$= 130$
$c = \sqrt{130}$

Vertices: $(0, \pm 9)$ Foci: $(0, \pm\sqrt{130})$

The center is (0,0) and the branches "open" in the y-direction.

Length of Transverse Axis = $2(9)$
$= 18$

Slopes of Asymptotes: m = $\pm\frac{9}{7}$