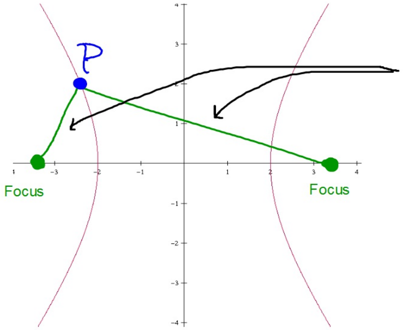
Sec 10-5 Hyperbolas:

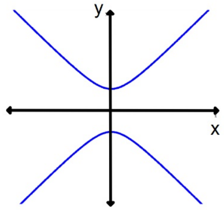
Definition: Set of all points such that the difference in the distance from any point to two fixed points, called the foci, is constant.

Like Ellipses
Foci are
"inside" the two
branches
of the Hyperbola

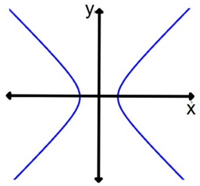


The difference between these two distances is the same no matter where pt P is on the Hyperbola.

Standard Form for the equation of a hyperbola whose center is at the origin:



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

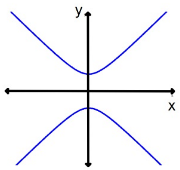


$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The direction the branches of the Hyperbola open is determined by....

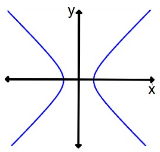
The variable in the numerator of the POSITIVE ratio.

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



y is in the numerator of the positive ratio, therefore, the braches open in the y direction.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

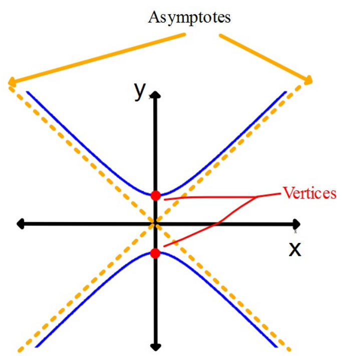


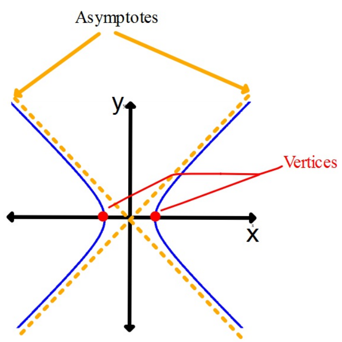
x is in the numerator of the positive ratio, therefore, the braches open in the x direction.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

a^2 is always the denominator of the positive ratio.

These graphs each have two asymptotes.

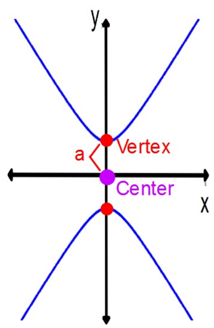




a is the distance from the center to each Vertex

What is b?

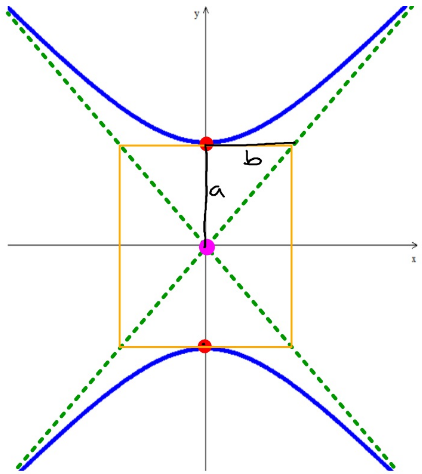
It is a value to help find
the slope of the Asymptotes.
See the next two screens.



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Asymptotes:

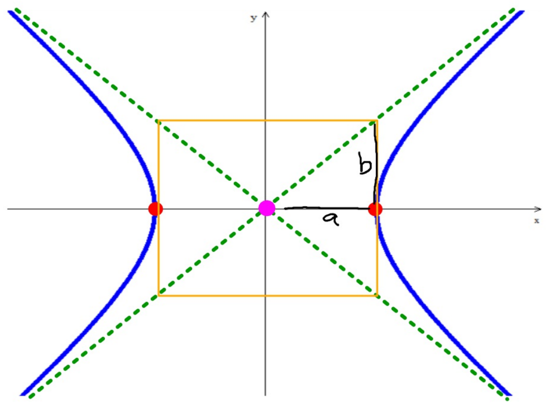
$$y = \pm \frac{a}{b} x$$

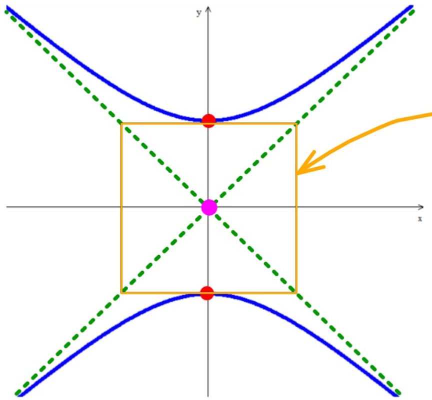


$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Asymptotes:

$$y = \pm \frac{b}{a} x$$





Our book refers to this as the

Central Rectangle

New vocabulary for a Hyperbola:

Transverse Axis: Segment connecting the Vertices.

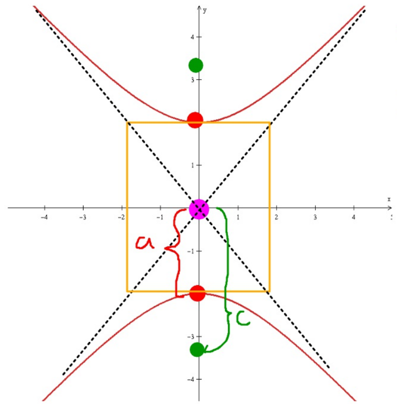
Length of the Transverse Axis = 2a

Just like in an Ellipse:

a is the distance from the center to a Vertex

c is the distance from the center to a Focus

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



Center: (0,0)

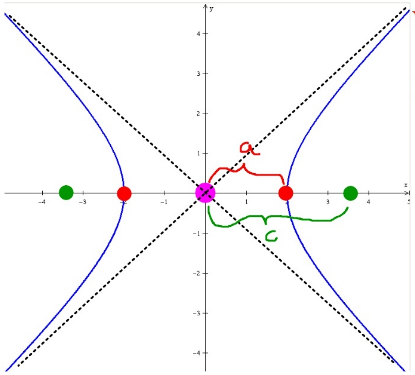
Vertices: (0,±a)

Foci: (0,±c)

Transverse Axis: Vertical
Length: 2a units

Asymptotes: $y = \pm\frac{a}{b}x$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Center: (0,0)

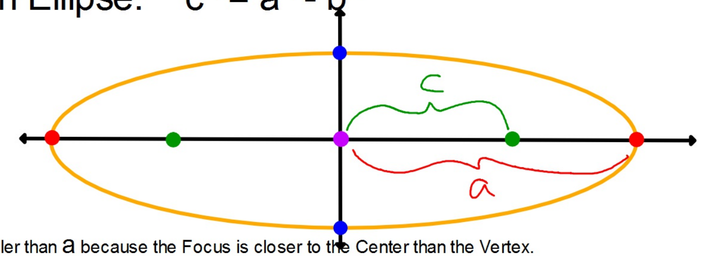
Vertices: $(\pm a,0)$

Foci: $(\pm c,0)$

Transverse Axis: Horizontal
Length: 2a units

Asymptotes: $y = \pm\frac{b}{a}x$

In an Ellipse: $c^2 = a^2 - b^2$



C is smaller than a because the Focus is closer to the Center than the Vertex.

Therefore, C^2 is smaller than a^2 so it makes sense to subtract b^2 from a^2.

In a Hyperbola: Relationship between a^2, b^2, and c^2:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$c^2=a^2+b^2$$

C is bigger than a because the Focus is farther away from the Center than the Vertex.

Therefore, C^2 is bigger than a^2 so it makes sense to add b^2 to a^2.