Transformations of Ellipses:

We will:
- Translate Ellipses

We won't:
- Reflect Ellipses over either axis
- Stretch or shrink them

The keys to all of these problems will be:

1. The coordinates of the CENTER.
   Everthing will will be measured from the center.

2. **a** Distance from Center to Vertices

   b Distance from Center to Co-Vertices

   c Distance from Center to Foci

Translating Ellipses:

Major Axis: Horizontal

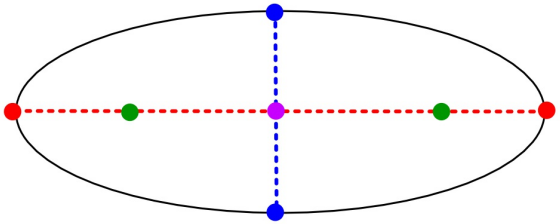
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Center: (h,k)

Vertices: $(h+a,k)$ $(h-a,k)$

Co-Vertices: $(h,k+b)$ $(h,k-b)$

Foci: $(h+c,k)$ $(h-c,k)$



Major Axis: Vertical

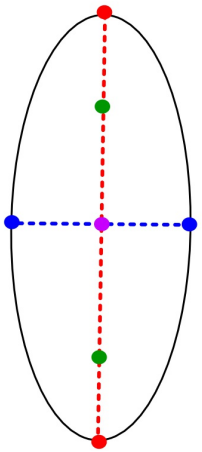
$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Center: (h,k)

Vertices: $(h,k+a)$ $(h,k-a)$

Co-Vertices: $(h+b,k)$ $(h-b,k)$

Foci: $(h,k+c)$ $(h,k-c)$



Use this equation to find the following coordinates:

$$\frac{(x+2)^2}{25}+\frac{(y-4)^2}{49}=1$$

(2 Left, 4 up)

Center: $(-2,4)$

Vertices: $(-2,11)$ $(-2,-3)$

Co-Vertices: $(3,4)$ $(-7,4)$

Foci: $(-2,4+2\sqrt{6})$ $(-2,4-2\sqrt{6})$ or $(-2,4\pm2\sqrt{6})$

major axis is vertical

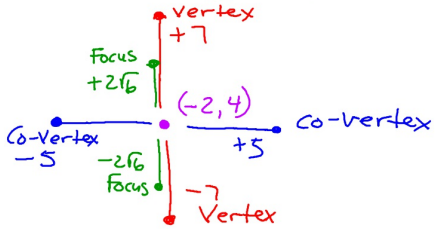
$a^2=49 \rightarrow a=\pm7$

minor axis is horizontal

$b^2=25 \rightarrow b=\pm5$

$c^2=49-25=24$

$c=\pm\sqrt{24}=\pm2\sqrt{6}$



Use this equation to find the following coordinates:

$$\frac{(x-1)^2}{100}+\frac{(y+3)^2}{64}=1$$

(1 RIGHT, 3 down)

Center: $(1,-3)$

Vertices: $(11,-3)$ $(-9,-3)$

Co-Vertices: $(1,2)$ $(1,-8)$

Foci: $(7,-3)$ $(-5,-3)$

major axis is horizontal

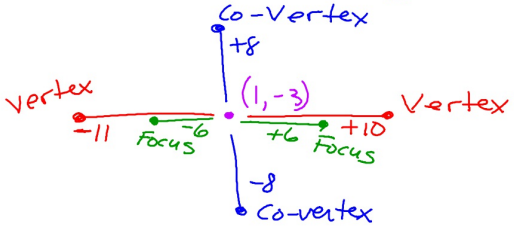
$a^2=100 \rightarrow a=\pm10$

minor axis is vertical

$b^2=64 \rightarrow b=\pm8$

$c^2=100-64=36$

$c=\pm6$



Write the equation of this Ellipse:

Center is (-9,11)

Major Axis is horizontal with a length of 26

Minor Axis length is 12

$2a=26$

$a=13$

$a^2=169$

$2b=12$

$b=6$

$b^2=36$

$$\frac{(x+9)^2}{169}+\frac{(y-11)^2}{36}=1$$

Write the equation of this Ellipse:

Vertices are (-7,11) and (-7,-7) (Vertical major axis)

Co-Vertices are (-12,2) and (-2,2) (horiz minor axis)

The center is the midpoint of the Vertices or midpoint of the Co-Vertices

Center $(-7,2)$

a = distance from $(-7,2)$ to $(-7,11)$ OR $(-7,2)$ to $(-7,-7)$

$a=9$

$a^2=81$

b = distance from $(-7,2)$ to $(-12,2)$ OR $(-7,2)$ to $(-2,2)$

$b=5$

$b^2=25$

$$\frac{(x+7)^2}{25}+\frac{(y-2)^2}{81}=1$$

Write the equation of this Ellipse:

Foci: (-7,-3) and (9,-3) → center is midpoint of foci (1,-3)

major axis is horiz → a^2 is under x^2

Minor axis length is 14

$2b = 14$

$b = 7$

$b^2 = 49$

c is distance from (1,-3) to either (-7,-3) or (9,-3)

$c = 8 \rightarrow c^2 = 64$

$c^2 = a^2 - b^2$ $\quad 64 = a^2 - 49$

$a^2 = 113$

$$\frac{(x-1)^2}{113} + \frac{(y+3)^2}{49} = 1$$

Write the equation of this Ellipse:

A Vertex: (12,5)

Foci: (12, 9) and 12, 17) → center is midpoint of foci (12, 13)

Foci are on a Vertical line so major axis is Vertical and a^2 will be under y^2.

a = distance from (12,13) to (12,5)

$a = 8 \rightarrow a^2 = 64$

c = distance from (12,13) to (12,9) or to (12,17)

$c = 4 \rightarrow c^2 = 16$

$c^2 = a^2 - b^2$

$16 = 64 - b^2 \rightarrow b^2 = 48$

$$\frac{(x-12)^2}{48} + \frac{(y-13)^2}{64} = 1$$