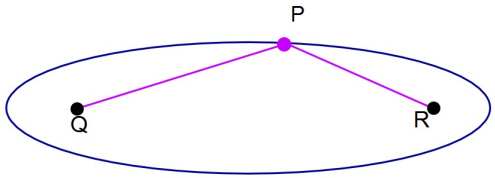
Section 10-4:

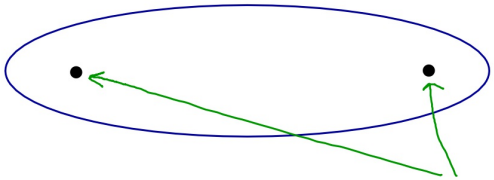
Ellipse

The set of all points in a plane such that the sum of the distances from two fixed points is a constant.



The points Q and R are the fixed points. P is a point on the Ellipse.
PQ + PR is a constant for any location of pt P.

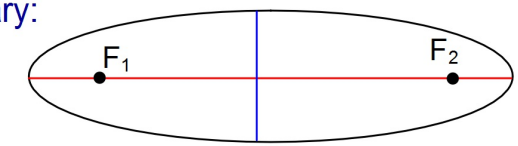
Ellipse Vocabulary:



The two fixed points are called the foci.

Each of these points is called a focus.

Ellipse Vocabulary:



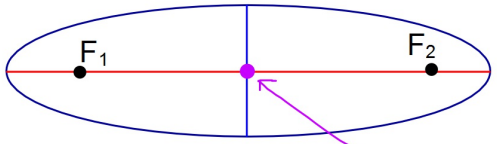
Major Axis:
The segment that connects two points on the ellipse and contains the foci.

Minor Axis:
The segment that connects two points on the ellipse, is perpendicular to the Major Axis, and passes through the midpoint of the Major Axis.

Major Axis: The longer of the two axes.

Minor Axis: The shorter of the two axes.

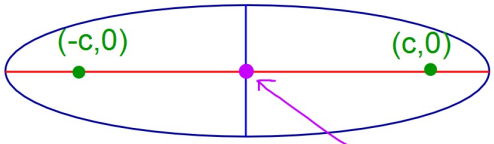
Ellipse Vocabulary:



Center of an Ellipse: Center

- Point of intersection of the two axes.
- Midpoint of the Major and/or Minor Axes.

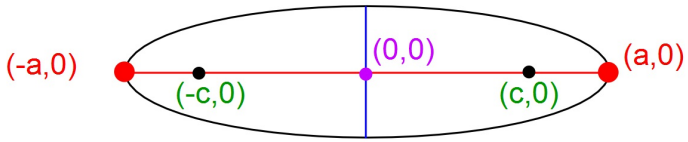
Ellipse Vocabulary:



Foci of an Ellipse: Center

- On the Major Axis.
- C units away from the center in both directions.

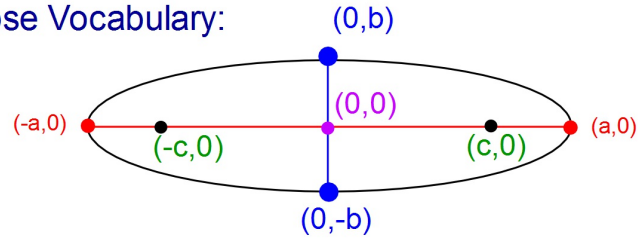
Ellipse Vocabulary:



Vertices of an Ellipse:
The endpoints of the Major Axis.

Labelled using the letter a

Ellipse Vocabulary:



Co-Vertices of an Ellipse: The endpoints of the minor axis labeled using the letter b

Standard Form of the Equations of an Ellipse with center at the origin.

$a^2 > b^2$

Horizontal Major Axis:

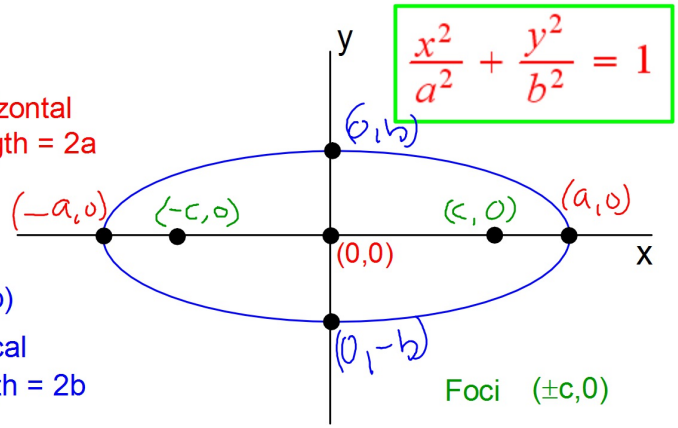
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Vertical Major Axis:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Vertices (±a,0)

Major Axis Horizontal
Length = 2a



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Co-Vertices (0,±b)

Minor Axis Vertical
Length = 2b

Foci (±c,0)

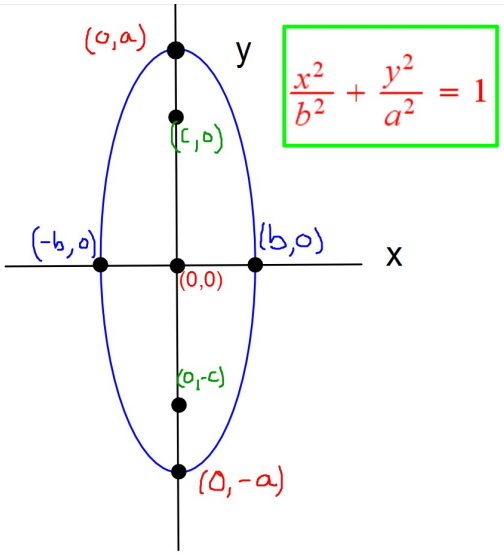
Vertices: (0,±a)

Major Axis Vertical
Length = 2a

Co-Vertices (±b,0)

Minor Axis Horizontal
Length = 2b

Foci (0,±c)



$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

The larger denominator is always a^2
and indicates the orientation of the Major Axis.

a Distance from Center to Vertices

b Distance from Center to Co-Vertices

c Distance from Center to Foci

Relationship between a, b, and c

$$c^2 = a^2 - b^2$$

Write the equation of the ellipse whose center is at the origin with the following Vertex and Co-Vertex:

Vertex (0,-9)

$a = 9$

$a^2 = 81$

in the y-direction

Co-Vertex (6,0)

$b = 6$

$b^2 = 36$

in the x-direction

$$\frac{x^2}{36} + \frac{y^2}{81} = 1$$

Write the equation of this ellipse.

The Major Axis is horizontal and has a length of 14 units. The Minor Axis is vertical and has a length of 8 units. The center is at the origin.

major axis length $14 = 2a$, $a = 7$, $a^2 = 49$ in the x direction

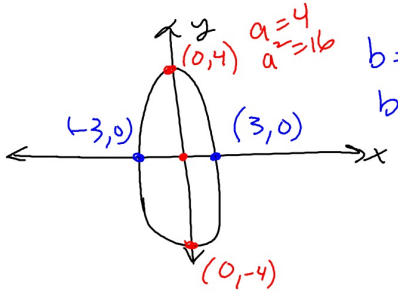
minor axis length $8 = 2b$, $4 = b$, $16 = b^2$ in the y direction

$$\frac{x^2}{49} + \frac{y^2}{16} = 1$$

Write the equation of the ellipse that is 8 units high and 6 units wide and has a center at the origin.

minor axis

major axis



$a = 4$, $a^2 = 16$

$b = 3$

$b^2 = 9$

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

Find the coordinates of the Vertices and Co-Vertices

$$\frac{x^2}{49} + \frac{y^2}{121} = 1$$

$b^2 = 49$ $\quad a^2 = 121$

$b = \pm 7$ $\quad a = \pm 11$

Vertices: $(0, \pm 11)$

Co-Vertices: $(\pm 7, 0)$

Find the coordinates of the Vertices and Co-Vertices

$$\frac{x^2}{81} + \frac{y^2}{36} = 1$$

$a^2 = 81$ $\quad b^2 = 36$

$a = \pm 9$ $\quad b = \pm 6$

Vertices: $(\pm 9, 0)$

Co-Vertices: $(0, \pm 6)$

Find the coordinates of the Foci of each Ellipse.

$$\frac{x^2}{49} + \frac{y^2}{121} = 1$$

b^2 $\quad a^2$

a^2 is under y^2 so major axis is vertical & foci are on the y-axis

$c^2 = a^2 - b^2$

$c^2 = 121 - 49$

$c^2 = 72$

$c = \pm\sqrt{72} = \pm 6\sqrt{2}$

Foci: $(0, \pm 6\sqrt{2})$

Find the coordinates of the Foci of each Ellipse.

$$\frac{x^2}{81} + \frac{y^2}{36} = 1$$

a^2 is under x^2 so major axis is horizontal & foci are on the x-axis

$a^2 = 81$

$b^2 = 36$

$c^2 = a^2 - b^2$

$c^2 = 81 - 36$

$c^2 = 45$

$c = \pm\sqrt{45} = \pm 3\sqrt{5}$

Foci: $(\pm 3\sqrt{5}, 0)$

Write the equation of the ellipse whose Vertices are (±10,0) and Foci are (±7,0)

$a = 10$

$a^2 = 100$

$c = 7$

$c^2 = 49$

$c^2 = a^2 - b^2$

$49 = 100 - b^2$

$b^2 = 51$

$$\frac{x^2}{100} + \frac{y^2}{51} = 1$$

Write the equation of the ellipse whose Co-Vertices are (±3,0) and Foci are (0,±9)

$b = 3$

$b^2 = 9$

This is in the x direction so put w/ x^2

$c = 9$

$c^2 = 81$

Since this is the y-direction foci & Vertices go with y

$c^2 = a^2 - b^2$

$81 = a^2 - 9$

$a^2 = 90$

$$\frac{x^2}{9} + \frac{y^2}{90} = 1$$

Find the coordinates of the vertices, co-vertices and foci of this ellipse.

$$12x^2 + 8y^2 = 72$$

$$\frac{12x^2}{72} + \frac{8y^2}{72} = \frac{72}{72}$$

$$\frac{x^2}{6} + \frac{y^2}{9} = 1$$

$b^2 = 6$

$b = \pm\sqrt{6}$

$a^2 = 9$

$a = \pm 3$

$c^2 = a^2 - b^2$

$c^2 = 9 - 6$

$c^2 = 3$

$c = \pm\sqrt{3}$

Vertices $(0, \pm 3)$

Co Vertices $(\pm\sqrt{6}, 0)$

Foci $(0, \pm\sqrt{3})$

You can now finish Hwk #23 Sec 10-4

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Problems 6, 7, 14, 20, 32, 37, 44, 47, 50, 59

Don't graph any ellipses