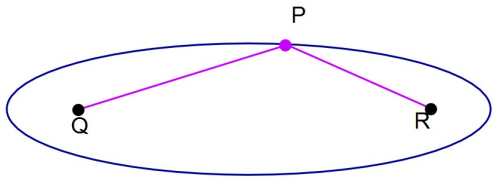
Section 10-4:

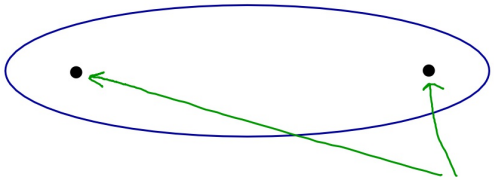
Ellipse

The set of all points in a plane such that the sum of the distances from two fixed points is a constant.



The points Q and R are the fixed points. P is a point on the Ellipse.
PM + PR is the same sum for any location of pt P as long as it is on the Ellipse.

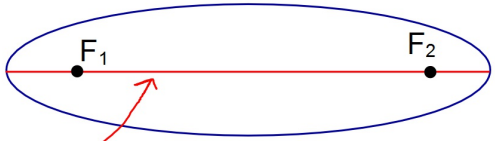
Ellipse Vocabulary:



The two fixed points are called the foci.

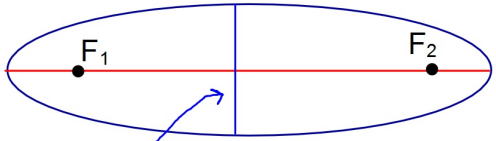
Each of these points is called a focus.

Ellipse Vocabulary:



Major Axis:
The segment that connects two points on the ellipse and contains the foci.

Ellipse Vocabulary:

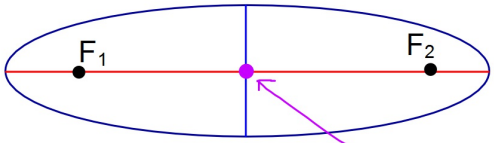


Minor Axis:
The segment that connects two points on the ellipse, is perpendicular to the Major Axis, and passes through the midpoint of the Major Axis.

Major Axis: The longer of the two axes.

Minor Axis: The shorter of the two axes.

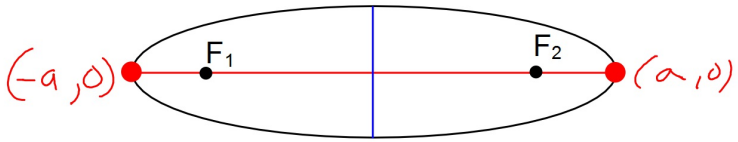
Ellipse Vocabulary:



Center of an Ellipse: Center

- Point of intersection of the two axes.
- Midpoint of the Major and/or Minor Axes.

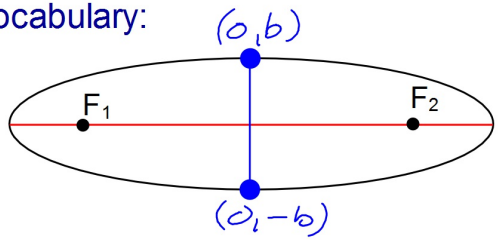
Ellipse Vocabulary:



Vertices of an Ellipse:
The endpoints of the Major Axis.

Labelled using the letter a

Ellipse Vocabulary:



Co-Vertices of an Ellipse: The endpoints of the minor axis labeled using the letter b

Standard Form of the Equation of an Ellipse with Horizontal Major Axis:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

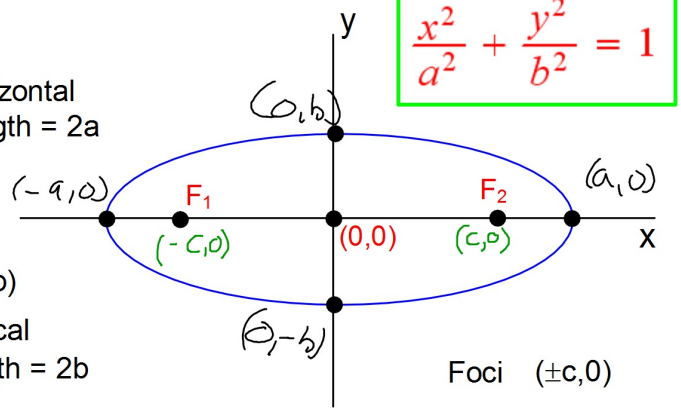
$a^2 > b^2$

The bigger denominator is under X.

Center (0,0)

Vertices (±a,0)

Major Axis Horizontal
Length = 2a



Co-Vertices (0,±b)

Minor Axis Vertical
Length = 2b

Foci (±c,0)

Standard Form of the Equation of an Ellipse with Vertical Major Axis:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

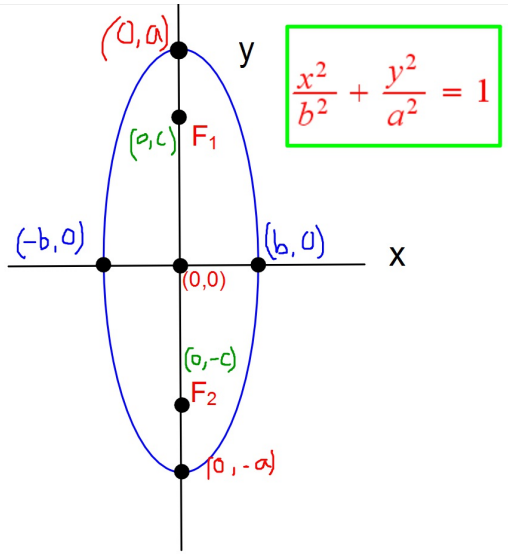
$a^2 > b^2$

The bigger denominator is under Y.

Center (0,0)

Vertices: (0,±a)

Major Axis Vertical
Length = 2a



Co-Vertices (±b,0)

Minor Axis Horizontal
Length = 2b

Foci (0,±c)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

The larger denominator is always a^2
and indicates the orientation of the Major Axis.

a Distance from Center to Vertices

b Distance from Center to Co-Vertices

c Distance from Center to Foci

Relationship between a, b, and c

$$c^2 = a^2 - b^2$$

Write the equation of the ellipse whose center is at the origin with the following Vertex and Co-Vertex:

Vertex (8,0)

$a = 8$
$a^2 = 64$

since this is an x-coordinate the Major Axis is horizontal (X)

Co-Vertex (0,-2)

$b = 2$
$b^2 = 4$

Since this is a y-coordinate the Minor Axis is vertical (Y)

$$\frac{x^2}{64} + \frac{y^2}{4} = 1$$

Write the equation of the ellipse that has Vertices at $(0,\pm 7)$ and Co-Vertices at $(\pm 3,0)$

$b = 3$
$b^2 = 9$

Since this is an x-coordinate the minor axis is horizontal (X)

$a = 7$
$a^2 = 49$

Since this is a y-coordinate the major axis is vertical (Y)

$$\frac{x^2}{9} + \frac{y^2}{49} = 1$$

Write the equation of this ellipse.

The Major Axis is horizontal and has a length of 30 units. The Minor Axis is vertical and has a length of 16 units. The center is at the origin.

$2a = 30$

$a = 15$

$a^2 = 225$

$16 = 2b$

$8 = b$

$b^2 = 64$

$$\frac{x^2}{225} + \frac{y^2}{64} = 1$$