

## Conic Sections Exploration

Refers to  
a Cone

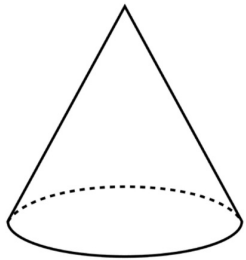
The surface created when slicing through a cone  
(a cross-section)

## The four Conic Sections:

- Circle
- Ellipse
- Hyperbola
- Parabola

Conic Sections are created by "slicing" a single cone or a double napped cone by a plane.

single cone



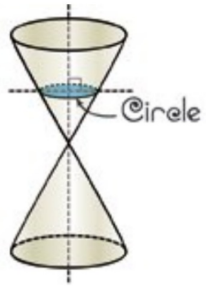
double napped cone



<http://illuminations.nctm.org/Activity.aspx?id=3506>

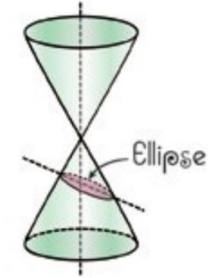
### Circles

If the plane intersects the cone perpendicular to the axis of the cone, then the curve produced will be a circle.



### Ellipses

If the plane intersects the cone at an angle greater than perpendicular to the axis but less than that of the line with the slope of the cone, the curve produced will be an ellipse.



### Parabola

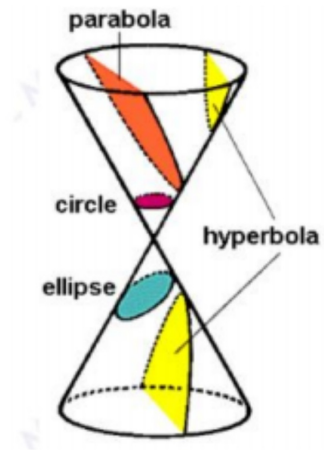
If the plane intersects the cone at the same angle as that of the line of the slope of the cone, then the curve produced will be a parabola.



### Hyperbola

If the plane intersects the cone at an angle greater than that of the line of the slope of the cone, then the curve produced will be a hyperbola.





For this exploration use this window on the Graphing Calculator:  
 $X: [-9.4, 9.4]$   $Y: [-6.2, 6.2]$

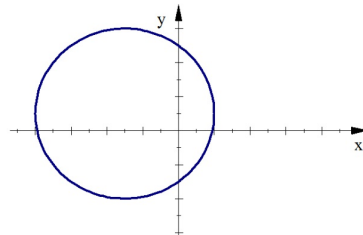
For each part do the following:

- Solve the equation for  $y$  then enter both equations into the graphing calculator.
- Sketch what you see in the calculator screen on the given axes.
- Describe or give a name to the graph.

**Part 1** EQ:  $(x + 3)^2 + (y - 1)^2 = 25$

a)  $y =$

$$y = \pm \sqrt{25 - (x+3)^2} + 1$$

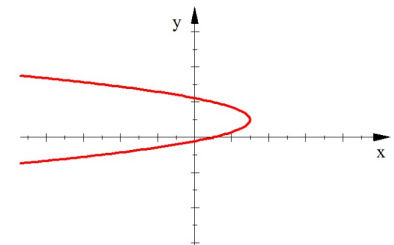


c) Name or Description **CIRCLE**

**Part 2** EQ:  $-2(y - 1)^2 + 3 = x$  b)

a)  $y =$

$$y = \pm \sqrt{\frac{x-3}{-2}} + 1$$



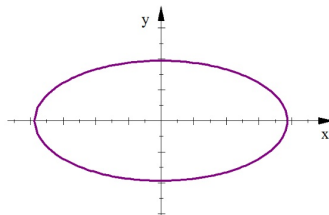
c) Name or Description **PARABOLA**

**Part 3** EQ:  $\frac{x^2}{60} + \frac{y^2}{15} = 1$

a)  $y =$

$$y = \pm \sqrt{15 \left( 1 - \frac{x^2}{60} \right)}$$

b)



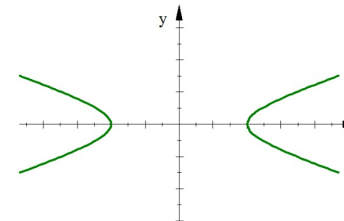
c) Name or Description **ELLIPSE**

**Part 4** EQ:  $\frac{x^2}{16} - \frac{y^2}{2} = 1$

a)  $y =$

$$y = \pm \sqrt{-2 \left( 1 - \frac{x^2}{16} \right)}$$

b)



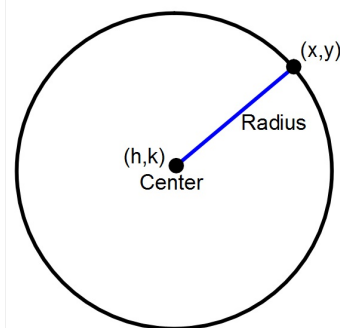
c) Name or Description **Hyperbola**

The distance formula:

The distance between the two points  $(x_1, y_1)$  &  $(x_2, y_2)$  is found by using the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Use the distance formula to find an expression for the length of the radius.



$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

If you square both sides:

$$(r)^2 = \left( \sqrt{(x-h)^2 + (y-k)^2} \right)^2$$

$$(x-h)^2 + (y-k)^2 = r^2$$

You get the equation of a circle whose center is  $(h,k)$  and whose radius is  $r \rightarrow \sqrt{r^2}$

Equation of a circle whose center is at the origin:  
and radius  $r$ .

$$x^2 + y^2 = r^2$$

Equation of a circle whose center is at the point  
 $(h,k)$ : and radius  $r$ .

$$(x - h)^2 + (y - k)^2 = r^2$$

Find the equation of the circle that is a translation of  $x^2 + y^2 = 16$   
9 units left and 3 units up.

center  
 $(-9, 3)$   
↑ ↑  
 $h \quad k$

$$(x + 9)^2 + (y - 3)^2 = 16$$