The Number

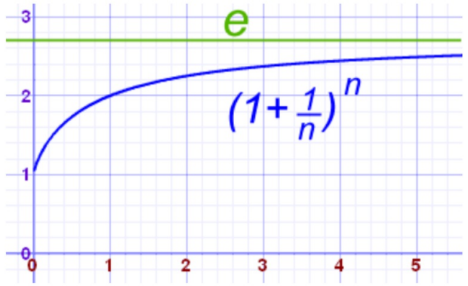
e

exploration

Frequency of compounding	#times per year compound interest (n)	$1\left(1+\frac{1}{n}\right)^{n}$	Dollar Value
Annually	$n = 1$	$1\left(1+\frac{1}{1}\right)^{1}$	2.00
Semiannually	$n = 2$	$1\left(1+\frac{1}{2}\right)^{2}$	2.25
quarterly	$n = 4$	$1\left(1+\frac{1}{4}\right)^{4}$	2.441
monthly	$n = 12$		2.613
weekly	$n = 52$		2.693
daily	$n = 365$		2.715
hourly	$n = 8760$		2.718
every minute	$n = 525{,}600$		2.718
every second	$n = 31{,}536{,}000$		2.718

As n gets larger the value of this equation approaches the value 2.718. This means the graph of this equation must have a Horizontal Asymptote.

the value of $(1 + 1/n)^n$ approaches e as n gets bigger and bigger:



n	$(1 + 1/n)^n$
1	2.00000
2	2.25000
5	2.48832
10	2.59374
100	2.70481
1,000	2.71692
10,000	2.71815
100,000	2.71827

Where is e used?

Like π, e is most often found in formulas.

e is called Euler's constant. It is named after:
Leonhard Euler: Swiss mathematician

Equation of a Catenary: $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right)$

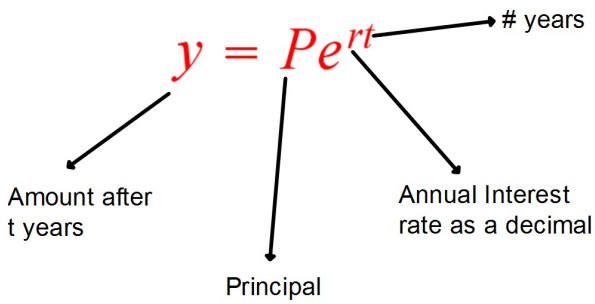
Catenary: A catenary is the shape that a cable assumes when it's supported at its ends and only acted on by its own weight. It is used extensively in construction, especially for suspension bridges

Famous Catenarys:

The more often interest is calculated the more money you will earn.

What is more often than every second?

Compounding Interest Continuously



You invest $20,000 in an account that pays 6% annual interest compounded CONTINUOUSLY. How much would you have after 25 years?

$y = Pe^{rt}$

$20{,}000\, e^{.06 * 25}$

$= 89{,}633.78$

Invest $20,000 at 6% annual interest for 25 years:

with Simple Interest you'll have $50,000

Compounding annually you'll have $85,837.41

Compounding monthly you'll have $89,299.39

Compounding continuously you'll have $89,633.78

You can now finish Hwk #15

Practice Sheet Sec 8-2

The value of an investment is increasing 8% each year. If the investment's value today is $125,000. Write an equation to model this situation.

Find the number of years, to the nearest hundredth, it will take to reach $1,000,000.

$1{,}000{,}000 = 125{,}000\left(1 + \frac{.08}{1}\right)^{t \cdot 1}$

$\frac{1{,}000{,}000}{125{,}000} = \frac{125{,}000\,(1.08)^t}{125{,}000}$

$8 = 1.08^t$

At this point using trial and error (easiest if using a graphing calculator)

OR

graphing the two sides of the equation separately and finding their point of intersection

will lead to this solution: t=27.02 years

You invest $20,000 in an account that pays 6% annual interest compounded CONTINUOUSLY. When would you have $150,000?

$y = Pe^{rt}$

$\frac{150{,}000}{20{,}000} = \frac{20{,}000\,e^{.06 \cdot t}}{20{,}000}$

$7.5 = e^{.06t}$

At this point using trial and error (easiest if using a graphing calculator)

OR

graphing the two sides of the equation separately and finding their point of intersection

will lead to this solution: t=33.58 years

Find the value of x in each equation:
Round to the nearest hundredth when needed.

1. $12x = 600$ $\quad \frac{12x}{12} = \frac{600}{12}$ $\quad x = 50$
2. $\sqrt[3]{64} = \sqrt[3]{x^3}$ $\quad x = 4$
3. $10^5 = x$ $\quad x = 100{,}000$
4. $10^x = 200$ this is going to take something new to solve for x!