Strategies for Simplifying Expressions

1) Change the expression into sines and cosines.
2) Look to use known formulas for purposes of substitution.
3) If there are fractions, gain a common denominator.
4) Use algebraic manipulations, like factoring, distributing, ...
5) If a strategy or substitution proves not to help, try something different.

Some tools available to you:

$$Tan\theta = \frac{Sin\theta}{Cos\theta} \qquad Cot\theta = \frac{1}{Tan\theta} = \frac{Cos\theta}{Sin\theta}$$

$$Csc\theta = \frac{1}{Sin\theta} \qquad Sec\theta = \frac{1}{Cos\theta}$$

Pythagorean Identities:

$Sin^2\theta + Cos^2\theta = 1$

$$Sin^2\theta = 1 - Cos^2\theta \qquad Cos^2\theta = 1 - Sin^2\theta$$

Starting with $Sin^2x + Cos^2x = 1$

Two other Pythagorean Identities can be derived:

Divide both sides by Sin^2x

$$\frac{Sin^2x}{Sin^2x} + \frac{Cos^2x}{Sin^2x} = \frac{1}{Sin^2x}$$

$$1 + COT^2x = CSC^2x$$

Divide both sides by Cos^2x

$$\frac{Sin^2x}{Cos^2} + \frac{Cos^2x}{Cos^2} = \frac{1}{Cos^2}$$

$$Tan^2x + 1 = Sec^2x$$

Trigonometric Tools:

Basic Identities:

$$\text{Tan}\theta = \frac{\text{Sin}\theta}{\text{Cos}\theta}$$

$$\text{Cot}\theta = \frac{1}{\text{Tan}\theta} = \frac{\text{Cos}\theta}{\text{Sin}\theta}$$

$$\text{Csc} = \frac{1}{\text{Sin}\theta}$$

$$\text{Sec} = \frac{1}{\text{Cos}\theta}$$

Pythagorean Identities:

$$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$$

$$\text{Sin}^2\theta = 1 - \text{Cos}^2\theta$$

$$\text{Cos}^2\theta = 1 - \text{Sin}^2\theta$$

$$\text{Tan}^2\theta + 1 = \text{Sec}^2\theta$$

$$1 + \text{Cot}^2\theta = \text{Csc}^2\theta$$

Simplify this trig expression:

$$\text{Csc}x(1 - \text{Cos}^2x)$$

$$\frac{1}{\sin x}(\sin^2 x) = \sin x$$

Simplify this trig expression: Two options are shown

$$\frac{\text{Sin}^2\theta}{\text{Cos}\theta} + \text{Cos}\theta$$

$$\frac{1-\cos^2}{\cos} + \cos$$

$$\frac{1}{\cos} - \cos + \cos$$

$$\frac{1}{\cos} = \sec\theta$$

$$\frac{\text{Sin}^2\theta}{\text{Cos}\theta} + \frac{\text{Cos}\theta}{1} \cdot \frac{\cos}{\cos}$$

$$\frac{\sin^2 + \cos^2}{\cos}$$

$$\frac{1}{\cos} = \sec\theta$$

Simplify this trig expression: Two options are shown

$$\text{Cot}\theta(\text{Tan}\theta + \text{Cot}\theta)$$

$$1 + \cot^2\theta$$

$$= \csc^2\theta$$

$$\text{Cot}\theta(\text{Tan}\theta + \text{Cot}\theta)$$

$$\frac{\cos}{\sin}\left(\frac{\sin}{\cos} + \frac{\cos}{\sin}\right)$$

$$1 + \frac{\cos^2}{\sin^2}$$

$$\frac{\sin^2}{\sin^2} + \frac{\cos^2}{\sin^2}$$

$$\frac{\sin^2 + \cos^2}{\sin^2}$$

$$\frac{1}{\sin^2} = \csc^2\theta$$

Simplify this trig expression:

$$\frac{\text{Sin}x}{1 - \text{Cos}^2 x} = \frac{\sin}{\sin^2} = \frac{1}{\sin} = \boxed{\csc x}$$

Simplify this trig expression: two possible options are shown

$$\frac{\text{Sec}x - \text{Cos}x}{\text{Sec}x}$$

$$\frac{\sec}{\sec} - \frac{\cos}{\sec}$$

$$1 - \frac{\cos}{\frac{1}{\cos}}$$

$$1 - \cos \cdot \frac{\cos}{1}$$

$$1 - \cos^2 = \boxed{\sin^2 x}$$

$$\frac{\text{Sec}x - \text{Cos}x}{\text{Sec}x}$$

$$\frac{\left(\frac{1}{\cos} - \cos\right)}{\frac{1}{\cos}} \cdot \frac{\cos}{\cos}$$

$$\frac{1 - \cos^2}{1}$$

$$1 - \cos^2 = \boxed{\sin^2 x}$$

Simplify this trig expression:

$\text{Sin}\theta + \text{Cos}\theta\text{Cot}\theta$

$$\sin + \cos \cdot \frac{\cos}{\sin}$$

$$\frac{\sin}{\sin} \cdot \frac{\sin}{1} + \frac{\cos^2}{\sin}$$

$$\frac{\sin^2 + \cos^2}{\sin} = \frac{1}{\sin} = \boxed{\csc x}$$

Simplify this trig expression:

$\text{Sin}x(1 + \text{Cot}^2 x)$

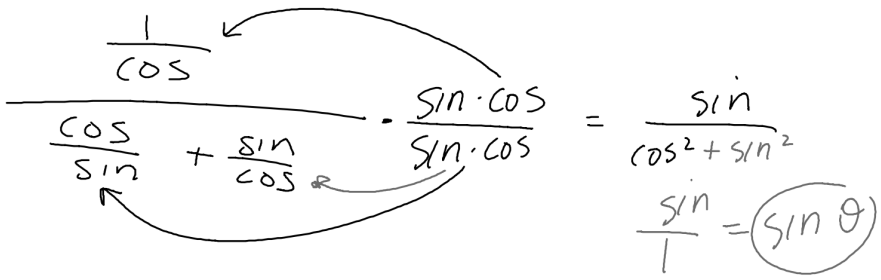
$$= \sin x\left(\csc^2 x\right)$$

$$= \sin\left(\frac{1}{\sin^2}\right)$$

$$= \frac{1}{\sin} = \boxed{\csc x}$$

Simplify this trig expression:

$$\frac{\sec\theta}{\cot\theta + \tan\theta}$$



Verifying trig identities:

Showing that the two sides of the equation really are equal.

Two basic techniques:

- Work on one side only and make it look like the other side.

or

- Work on both sides until they look the same.

When trying to verify/prove an identity you CAN'T do the following:

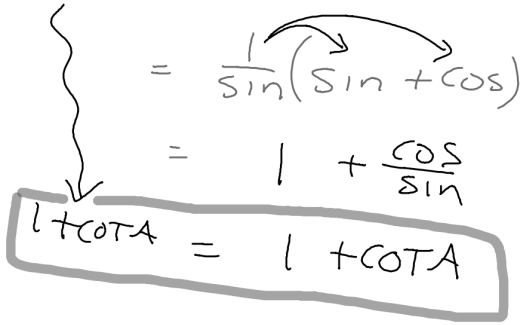
- Move terms from one side of the equation to the other side
- Multiply/divide/square/square root both sides of the equation
- Add/Subtract from both sides of the equation

You are NOT solving

You are trying to show the two sides of the equation are equal which means you don't know they are equal....YET.

Verify this identity:

$1 + \cot A = \csc A(\sin A + \cos A)$



Your job is done when you've showed that the two sides are Identical.