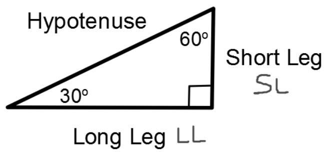
Special Right Triangles:

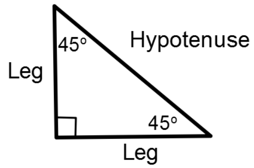
30° - 60° - 90°



Short Leg = $\frac{1}{2} \cdot$ HYPOT

Long Leg = $SL \cdot \sqrt{3}$

45° - 45° - 90°

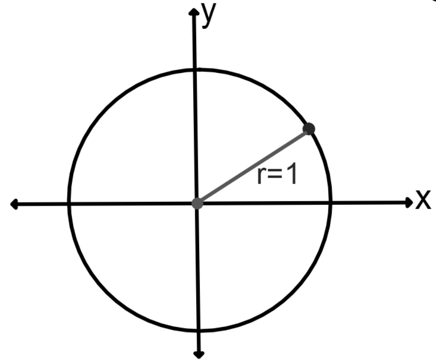


Legs are equal

Hypotenuse = $LEG \cdot \sqrt{2}$

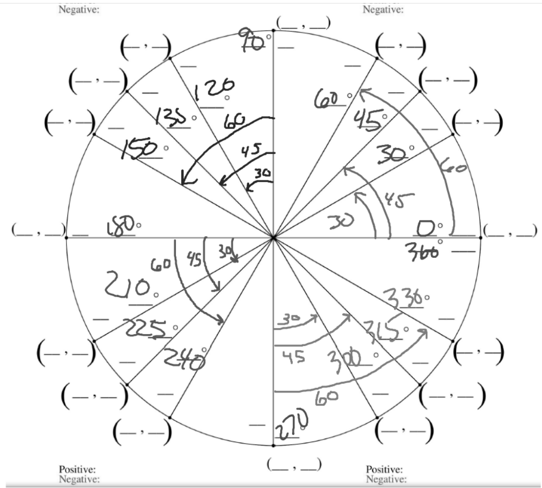
The Unit Circle:

A circle whose center is at the origin and has radius = 1.



The unit circle is used to find the exact value for Sinθ and Cosθ using the special right triangles.

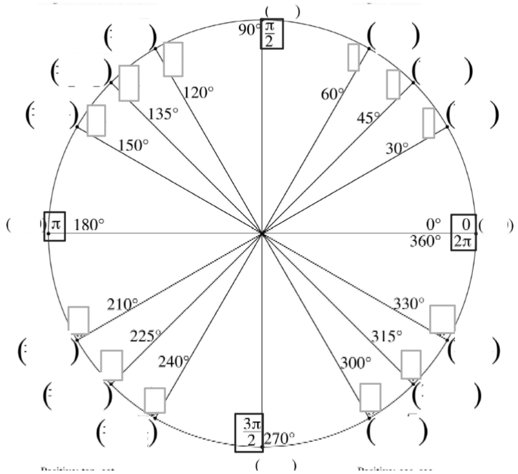
So all the angles on the unit circle are related to either 30°, 60°, or 45°



fill in all the angles in degrees

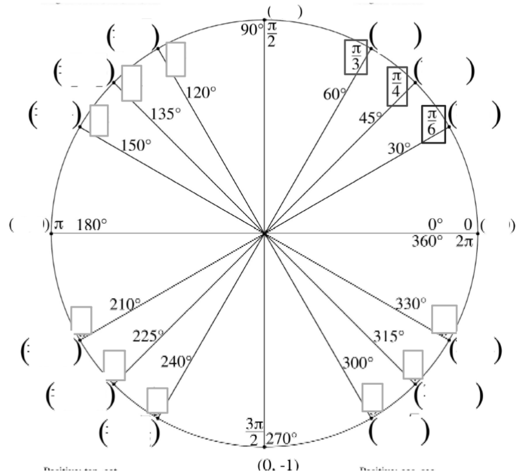
You can start with the angle measures on the axes.

then in each quadrant the angles are 30, 45, and 60 more than the axes.



fill in all the angles in radians.

You can start with the radians on the axes.



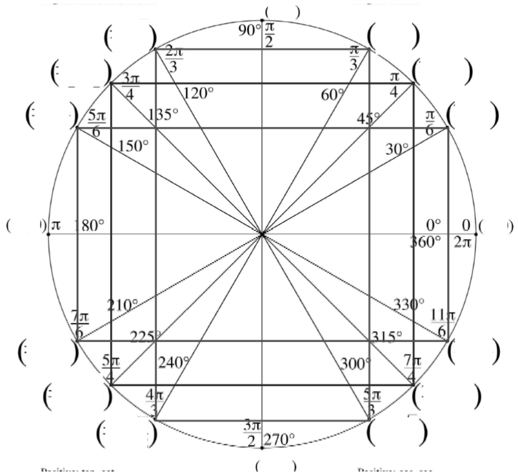
fill in all the angles in radians.

You can then convert the 30° , 45° , and 60° angles in the first quadrant to radians.

$$30° \cdot \frac{\pi}{180°} = \frac{\pi}{6}$$

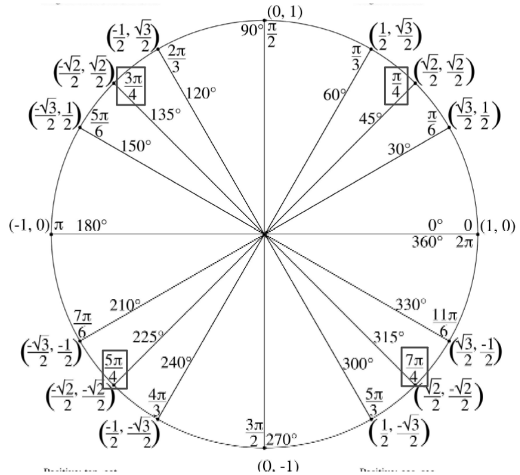
$$45° \cdot \frac{\pi}{180°} = \frac{\pi}{4}$$

$$60° \cdot \frac{\pi}{180°} = \frac{\pi}{6}$$

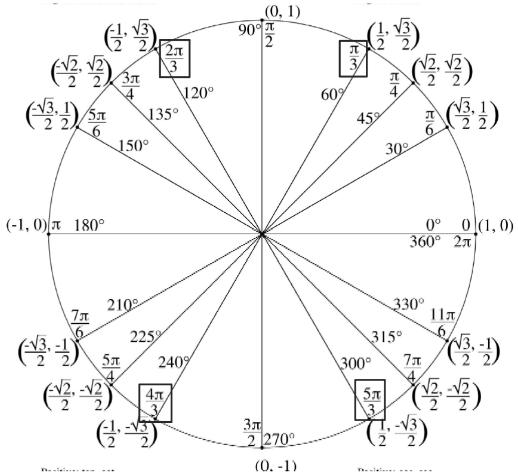


To fill out the remaining radian measures you can use many patterns and relationships. Some ideas are given on this and following pages.

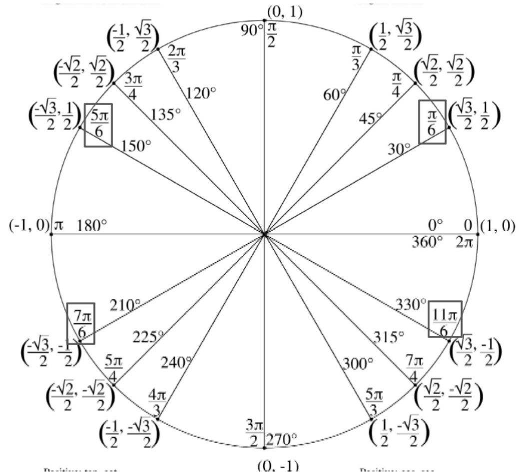
Notice the denominators at the corners of each rectangle are the same.



For the numerators of the π/4's notice they are a list of odd numbers.

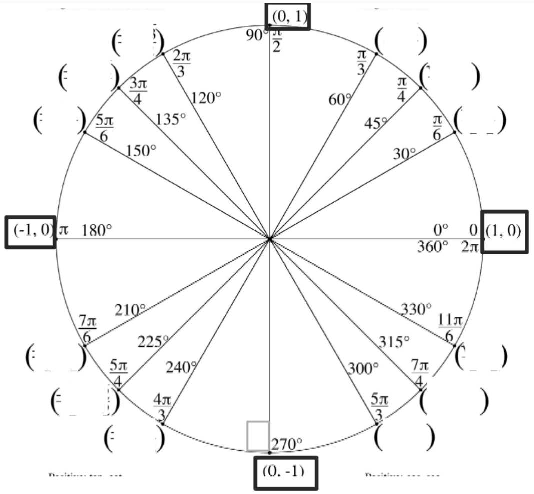


For the π/3's you can increase the numerator by one starting with π/3. But you skip 3π/3 because this is the same as just π.



For the π/6's you can increase the numerator by one starting with π/6 until you find one that doesn't reduce.

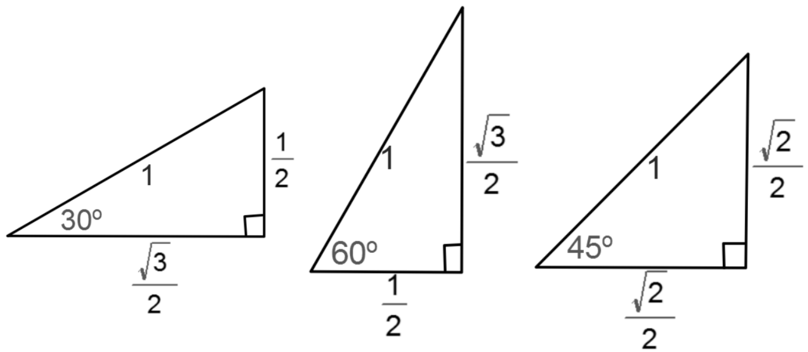
For example, after π/6 you skip 2π/6, 3π/6, and 4π/6 because they all reduce to something else.



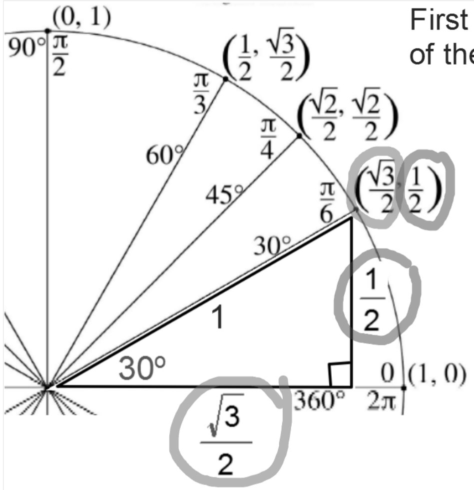
Fill in the coordinates on the axes.

Since this is the Unit Circle the radius is 1 and the points on the axes are just 1 unit right, up, left, and down from the origin.

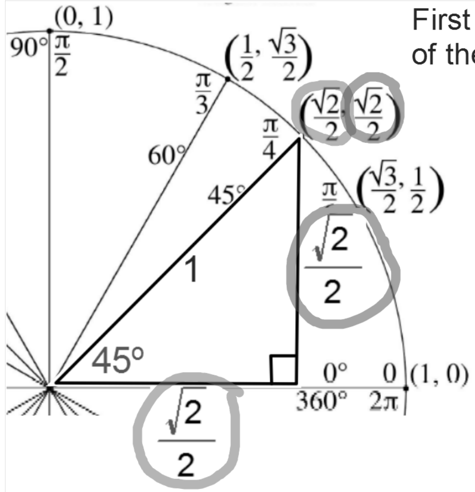
The Unit Circle involves the angles in Special Right Triangles which means it probably involves the sides too!



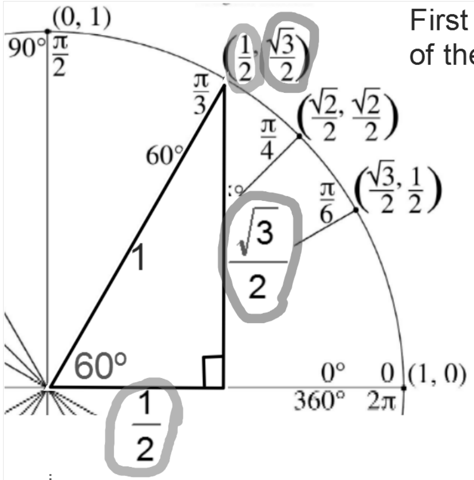
First Quadrant
of the Unit Circle



First Quadrant
of the Unit Circle

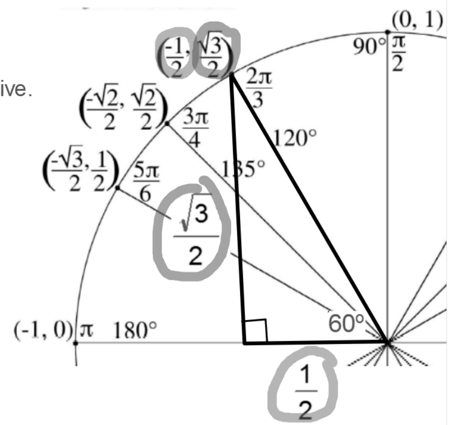


First Quadrant
of the Unit Circle



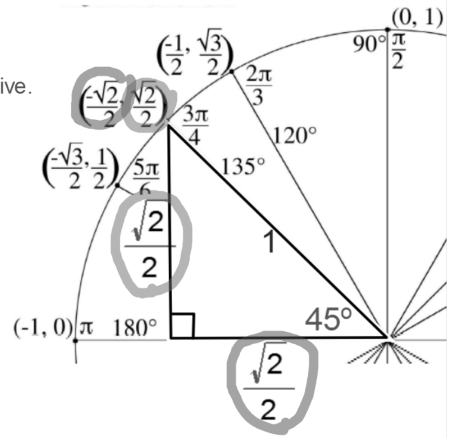
i

Second Quadrant
of the Unit Circle
x-coordinates are negative
and y-coordinates are positive.



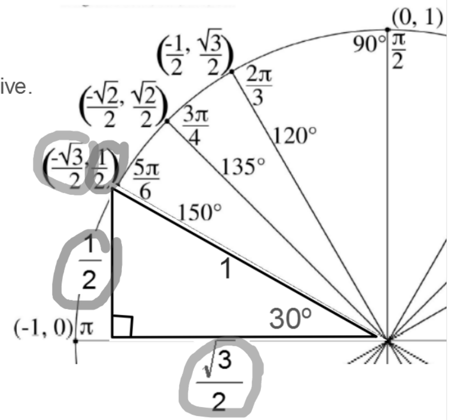
Second Quadrant of the Unit Circle

x-coordinates are negative and y-coordinates are positive.



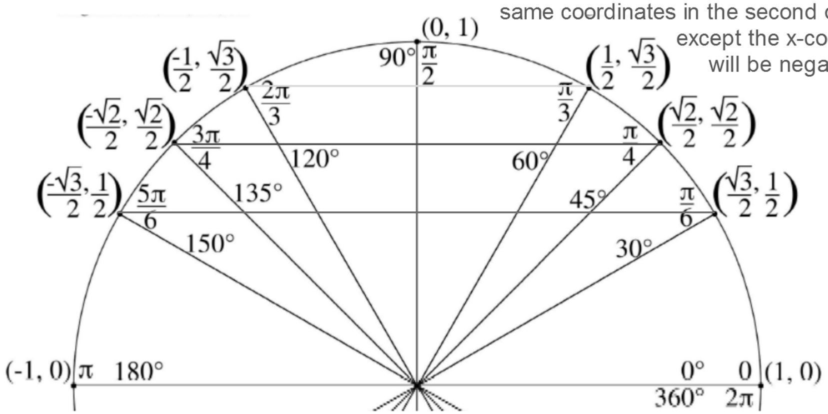
Second Quadrant of the Unit Circle

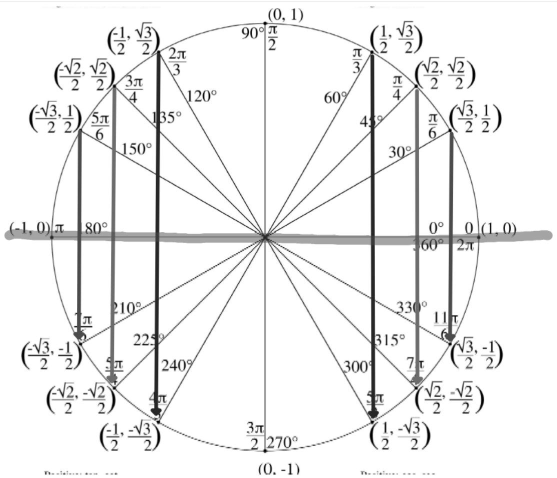
x-coordinates are negative and y-coordinates are positive.



What patterns do you notice?

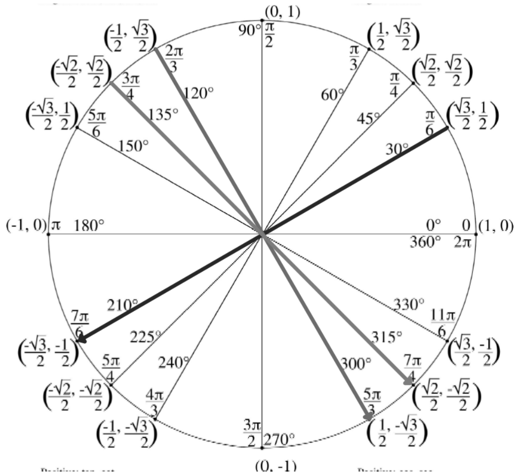
When you reflect points in the first quadrant over the y-axis you get the same coordinates in the second quadrant except the x-coordinate will be negative





To find the coordinates of the points in the third and fourth quadrants you can reflect the first and second quadrants over the x-axis.

But, in the third quadrant all coordinates are negative and in the fourth quadrant x is negative and y is positive.

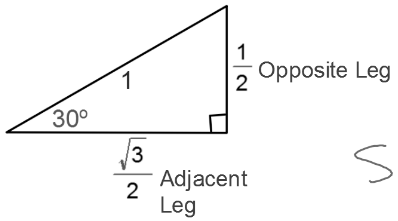


To find the coordinates of the points in the third and fourth quadrants you can take the points in the first and second quadrants and draw a diameter. At the end of the diameter you will find the same coordinates.

But, in the third quadrant all coordinates are negative and in the fourth quadrant x is negative and y is positive.

example diameters are shown.

There are many other patterns and relationships you can use to fill out the Unit Circle, only some of them were demonstrated in these notes.

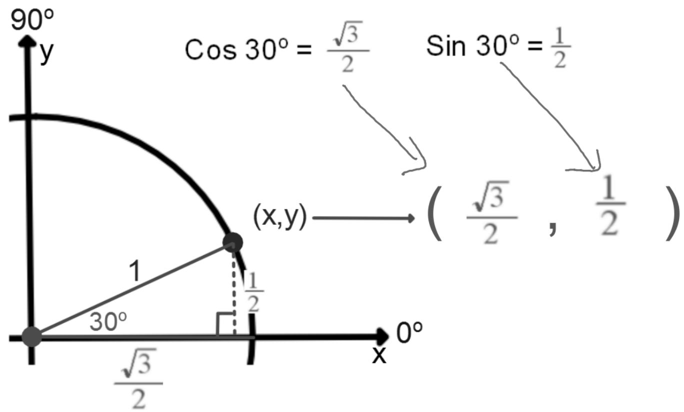


SOHCAHTOA

Find the exact value of each:

$\cos 30^\circ = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$ $\sin 30^\circ = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

What are the coordinates of the blue point?



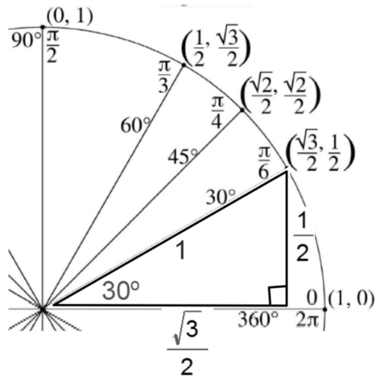
Coordinates on the Unit Circle:

$$(x, y) \longrightarrow (\cos\theta, \sin\theta)$$

Actually, Cosθ is defined as $\dfrac{x}{r}$

but since r=1: Cos$\theta = x$

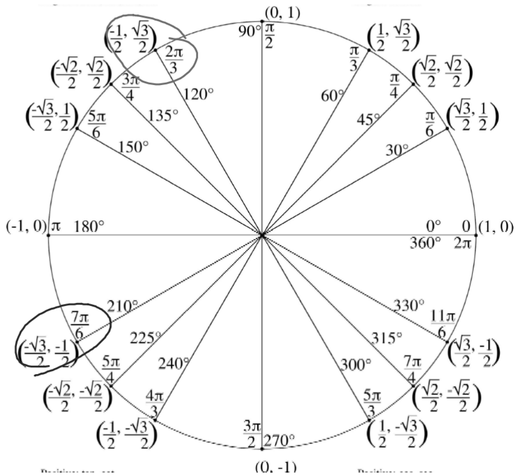
Finding Sinθ, Cosθ, and Tanθ using the Unit Circle



Cosθ = x (x-coordinate)

Sinθ = y (y-coordinate)

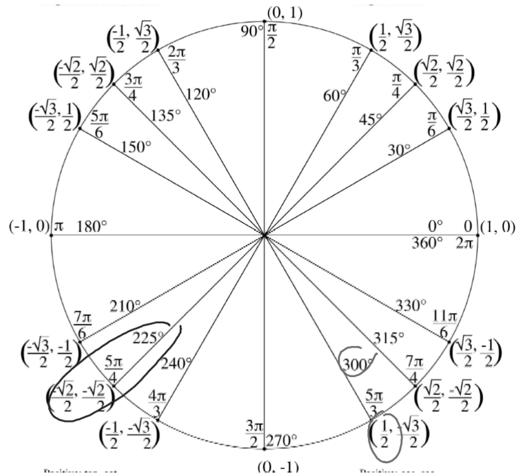
$$\text{Tan}\theta = \frac{\text{Opp Leg}}{\text{Adj Leg}} = \frac{y}{x}$$



Use the Unit Circle to find the EXACT value of each.

$\text{Cos}\,\frac{2\pi}{3} = -\frac{1}{2}$

$\text{Sin}\,\frac{7\pi}{6} = -\frac{1}{2}$



$\text{Sin}(-135°) = -\frac{\sqrt{2}}{2}$

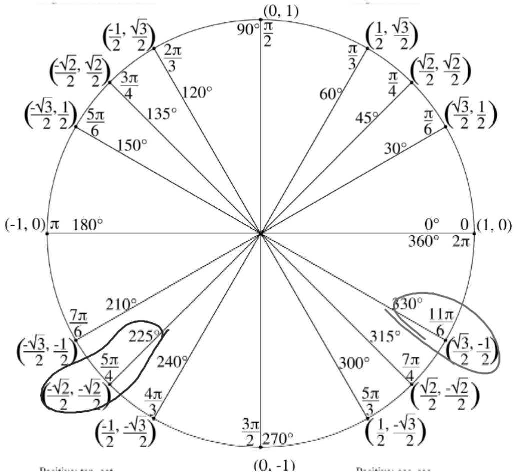
$+360$

$\sin(225°)$

$\text{Cos}\,660° = \frac{1}{2}$

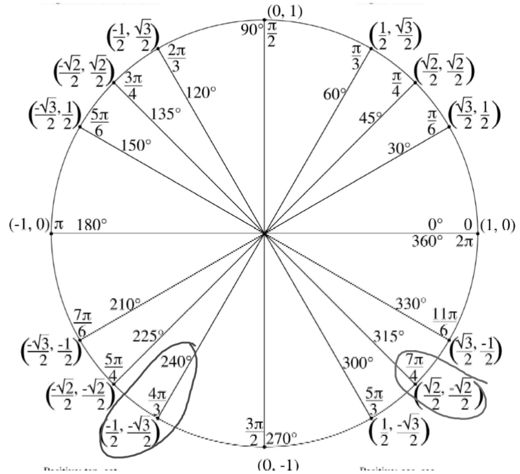
-360

$\cos(300°)$



$\text{Cos}\,\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$

$\text{Sin}\,225° = -\frac{\sqrt{2}}{2}$



$\text{Tan}(-\frac{\pi}{4}) = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$

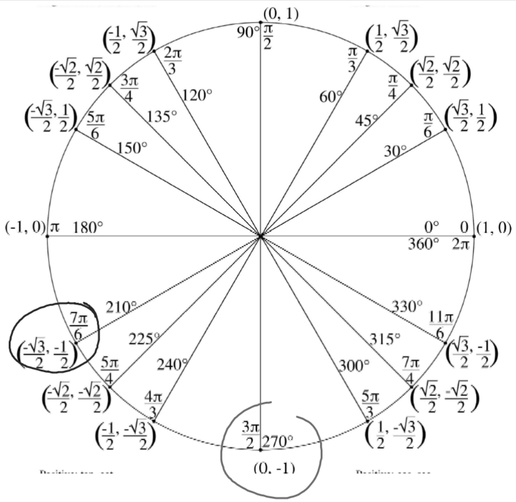
$+\frac{8\pi}{4}$

$\text{Tan}(\frac{7\pi}{4})$

$\text{Cos}(-480°) = -\frac{1}{2}$

$+720$

$\cos(240°)$



$\text{Sin}(-\frac{17\pi}{6}) = -\frac{1}{2}$

$+\frac{24\pi}{6}$

$\sin(\frac{7\pi}{6})$

$\text{Tan}\,630°$

-360

$\text{Tan}\,270° = \frac{-1}{0}$

undefined