Sec 8-4: Properties of Logarithms

Power Property: $\log_a C^b = b \cdot \log_a C$

The exponent becomes a coefficient.

Rewrite using the Power Property:

$\log W^4 + \log X^7 = 4\log W + 7\log X$

Solve without using the change of base formula. $8^x = 90$

$\log 8^x = \log 90$ take the log of both sides

$\frac{x \log 8}{\log 8} = \frac{\log 90}{\log 8}$ apply the power rule

divide by log8 to solve for x

$x = \frac{\log 90}{\log 8} = 2.16$

Product Property of Logarithms:

$\log_b MN = \log_b M + \log_b N$

Write as a single logarithm then solve:

$\log_5 x + \log_5 2 = 3$

$\log_5(2x) = 3$

$5^3 = 2x$

$125 = 2x$

$x = 62.5$

Quotient Property of Logarithms:

$\log_b \frac{M}{N} = \log_b M - \log_b N$

Write as a single logarithm then solve.

$\log_2 8 - \log_2 x = 5$

$\log_2\left(\frac{8}{x}\right) = 5$

$2^5 = \frac{8}{x}$

$32 = \frac{8}{x}$

$x = .25$

Properties	Properties of Logarithms

For any positive numbers, M, N, and b, $b \neq 1$,

$\log_b MN = \log_b M + \log_b N$	**Product Property**
$\log_b \frac{M}{N} = \log_b M - \log_b N$	**Quotient Property**
$\log_b M^x = x \log_b M$	**Power Property**

Use the Properties of Logarithms to write each as a single logarithm:

1. $3\log_4 K + 2\log_4 Q$

$= \log_4 K^3 + \log_4 Q^2$

$= \boxed{\log_4(K^3 Q^2)}$

2. $5\log R - 6\log X + \frac{1}{2}\log Y$

$= \underbrace{\log R^5 - \log x^6} + \log\sqrt{Y}$

$= \log \frac{R^5}{x^6} + \log\sqrt{y}$

$= \boxed{\log\left(\frac{R^5}{x^6} \cdot \sqrt{y}\right) \text{ or } \log \frac{R^5\sqrt{y}}{x^6}}$

3. $3\log_2 A - \frac{1}{3}\log_2 B - 4\log_2 C$

$\log_2 A^3 - \log_2 \sqrt[3]{B} - \log_2 C^4$

$= \boxed{\log_2\left(\frac{A^3}{\sqrt[3]{B}\ C^4}\right)}$

Solving Exponential Equations:

1. Isolate the exponential (b^x)
2. Change to a Logarithm
3. Solve for x, if necessary.

Exponential Equation

Range:

Domain: Any real number

$$y = b^x$$

$b>0,\ b\neq 1$

Since the exponent of an exponential equation can be any real number when we are solving for the exponent you won't have to worry about getting extraneous solutions. But checking your answer to make sure you didn't make a mistake is always a good idea.

Solve each exponential equation:

1. $7 + 4^{x+3} = 100$

$$-7 \qquad -7$$

$$4^{x+3} = 93$$

$$\log_4 93 = x+3$$

$$x = \frac{\log 93}{\log 4} - 3 = \boxed{.27}$$

2. $5 \cdot e^{2x-1} + 4 = 49$

$$-4 \qquad -4$$

$$\frac{5e^{2x-1}}{5} = \frac{45}{5}$$

$$e^{2x-1} = 9$$

$$\ln 9 = 2x-1$$

$$x = \frac{\ln 9 + 1}{2} \approx \boxed{1.6}$$