Sec 6-6: The Fundamental Theorem of Algebra

Theorem — **Fundamental Theorem of Algebra**

If $P(x)$ is a polynomial of degree $n \geq 1$ with complex coefficients, then $P(x) = 0$ has at least one complex root.

All polynomials that are Linear or above have at least one complex root (real or imaginary)

Corollary

Including complex roots and multiple roots, an nth degree polynomial equation has exactly n roots; the related polynomial function has exactly n zeros.

The degree of a polynomial determines the total number of roots, total of real and imaginary. Some roots may repeat which means a polynomial of degree n may not have n different roots but a total of n roots nonetheless.

For each polynomial:

- state the total number of complex roots
- state the possible number of real roots

Since imaginary numbers always come in pairs the number of real zeros always decreases by 2's

1. $f(x) = 7x^3 + 11x^2 - 2x + 13$

   Total # complex roots = 3 Possible # of real roots = 3, 1

2. $y = -9x^6 + x^5 - 3x^4 - 21x^3 + x^2 - 16x + 4$

   Total # complex roots = 6 Possible # of real roots = 6, 4, 2, 0

3. $f(x) = 5x^9 - 14x^7 - 4x^2 + 74x - 103$

   Total # complex roots = 9 Possible # of real roots = 9, 7, 5, 3, 1

Sec 6-8: The Binomial Theorem

Expand each. Write answers in Standard Form.

1. $(a + b)^0 =$

   $(a+b)^0 = \boxed{1}$

2. $(a + b)^1 =$

   $(a + b)^1 = \boxed{a + b}$

3. $(a + b)^2 =$

   $(a + b)^2 = \boxed{a^2 + 2ab + b^2}$

4. $(a + b)^3 =$

   $(a + b)^3 = \boxed{a^3 + 3a^2b + 3ab^2 + b^3}$

Find the next two rows of this pattern of numbers.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

this would be the next row

This pattern is called:
Pascal's Triangle.

Every row starts and ends with a 1. To find every other number you add the two numbers above that spot.

Row 0 → 1

Row 1 → 1 1

Row 2 → 1 2 1

Row 3 → 1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

The Row Number in Pascal's Triangle is the 2nd Number in each row.

Do you see a connection to Pascal's Triangle and the results of expanding powers of (a + b) ?

$(a+b)^0 = 1 \longrightarrow 1$

$(a+b)^1 = a+b \longrightarrow 1a + 1b$

$(a+b)^2 = a^2 + 2ab + b^2 \longrightarrow 1a^2 + 2ab + 1b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \longrightarrow 1a^3 + 3a^2b + 3ab^2 + 1b^3$

The coefficients of each term are the numbers in the row of Pascal's Triangle that to corresponds to the exponent of (a + b).

Do you notice a pattern in the exponents?

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b^1 + 6a^2b^2 + 4ab^3 + b^4$

Powers of a decrease from left to right

Powers of b decrease from right to left

For each term the powers of a and b add to the exponent of (a + b)

What do you notice about how many terms each has?

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

The number of terms is one more then the power on (a + b).

Expand this: $(a + b)^5$

$\underline{1a^5} + \underline{5a^4b} + \underline{10a^3b^2} + \underline{10a^2b^3} + \underline{5ab^4} + \underline{1b^5}$

Do you notice a pattern with the signs?

$$(a-b)^6 = a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$$

$$(a-b)^7 = a^7 - 7a^6b + 21a^5b^2 - 35a^4b^3 + 35a^3b^4 - 21a^2b^5 + 7ab^6 - b^7$$

when there is a minus in the middle of the binomial, the signs of the terms alternate. If a is positive then the first term is positive and the second term becomes negative.

If $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

How would this expansion be different? $(a - b)^4$

$(a - b)^4 = (a + -b)^4 = a^4 + 4a^3(-b) + 6a^2(-b)^2 + 4a(-b)^3 + (-b)^4$

$= a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

$(a - b)^4 = (a + -b)^4 = a^4 + 4a^3(-b) + 6a^2($

How would the results of these two be different?

$(a - b)^5$

This would start with a^5

The second term would be negative and alternate from there.

$(-a + b)^5$

This would start with $-a^5$

The second term would be positive and alternate from there.

How would the results of these two be different?

$(a - b)^4$

This would start with a^4

The second term would be negative and alternate from there.

$(-a + b)^4$

This would also start with a^4 because $(-a)^4$ is still positive.

The second term would be negative and alternate from there.