

## Sec 6-6: The Fundamental Theorem of Algebra

### Theorem

### Fundamental Theorem of Algebra

If  $P(x)$  is a polynomial of degree  $n \geq 1$  with complex coefficients, then  $P(x) = 0$  has at least one complex root.

All polynomials that are Linear or above have at least one complex root (real or imaginary)

### Corollary

Including complex roots and multiple roots, an  $n$ th degree polynomial equation has exactly  $n$  roots; the related polynomial function has exactly  $n$  zeros. The degree of a polynomial determines the total number of roots, total of real and imaginary. Some roots may repeat which means a polynomial of degree  $n$  may not have  $n$  different roots but a total of  $n$  roots nonetheless.

For each polynomial:

- state the total number of complex roots
- state the possible number of real roots

Since imaginary numbers always come in pairs the number of real zeros always decreases by 2's

1.  $f(x) = 7x^3 + 11x^2 - 2x + 13$

Total # complex roots = 3

Possible # of real roots = 3, 1

2.  $y = -9x^6 + x^5 - 3x^4 - 21x^3 + x^2 - 16x + 4$

Total # complex roots = 6

Possible # of real roots = 6, 4, 2, 0

3.  $f(x) = 5x^9 - 14x^7 - 4x^2 + 74x - 103$

Total # complex roots = 9

Possible # of real roots = 9, 7, 5, 3, 1

## Sec 6-8: The Binomial Theorem

Expand each. Write answers in Standard Form.

1.  $(a + b)^0 =$

$(a+b)^0 = 1$

2.  $(a + b)^1 =$

$(a + b)^1 = a + b$

3.  $(a + b)^2 =$

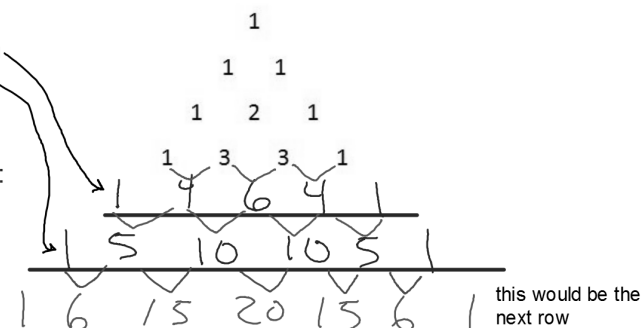
$(a + b)^2 = a^2 + 2ab + b^2$

4.  $(a + b)^3 =$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Find the next two rows of this pattern of numbers.

This pattern is called: Pascal's Triangle.



Every row starts and ends with a 1. To find every other number you add the two numbers above that spot.



Expand this:  $(a + b)^5$

$$\underline{1a^5} + \underline{5a^4b} + \underline{10a^3b^2} + \underline{10a^2b^3} + \underline{5ab^4} + \underline{1b^5}$$

Do you notice a pattern with the signs?

$$(a - b)^6 = a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$$

$$(a - b)^7 = a^7 - 7a^6b + 21a^5b^2 - 35a^4b^3 + 35a^3b^4 - 21a^2b^5 + 7ab^6 - b^7$$

when there is a minus in the middle of the binomial, the signs of the terms alternate. If a is positive then the first term is positive and the second term becomes negative.

$$\text{If } (a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

How would this expansion be different?  $(a - b)^4$

$$\begin{aligned}(a - b)^4 &= (a + -b)^4 = a^4 + 4a^3(-b) + 6a^2(-b)^2 + 4a(-b)^3 + (-b)^4 \\ &= a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4\end{aligned}$$

How would the results of these two be different?

$$(a - b)^5$$

This would start with

$$a^5$$

The second term would be negative and alternate from there.

$$(-a + b)^5$$

This would start with

$$-a^5$$

The second term would be positive and alternate from there.

$$(a - b)^4 = (a + -b)^4 = a^4 + 4a^3(-b) + 6a^2(-b)^2 + 4a(-b)^3 + (-b)^4$$

How would the results of these two be different?

$$(a - b)^4$$

This would start with  
 $a^4$

The second term would  
be negative and alternate  
from there.

$$(-a + b)^4$$

This would also start  
with  $a^4$  because  $(-a)^4$   
is still positive.

The second term would  
be negative and alternate  
from there.