Sec 6-6: The Fundamental Theorem of Algebra

Theorem

Fundamental Theorem of Algebra

If P(x) is a polynomial of degree $n \ge 1$ with complex coefficients, then P(x) = 0 has at least one complex root. All polynomials that are Linear or above have at least one complex root (real or imaginary)

Corollary

Including complex roots and multiple roots, an *n*th degree polynomial equation has exactly n roots; the related polynomial function has exactly n zeros. The degree of a polynomial determines the total number of roots, total of real and imaginary. Some roots may repeat which means a polynomial of degree n may not have n different roots but a total of n roots nonetheless.

Sec 6-8: The Binomial Theorem

Expand each. Write answers in Standard Form.

1.
$$(a + b)^0 =$$

$$(a+b)^0 = 1$$

$$(a+b)^0 = 1$$

3.
$$(a + b)^2 =$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

2.
$$(a + b)^1 =$$

$$(a + b)^1 = a + b$$

4.
$$(a + b)^3 =$$

$$(a + b)^2 =$$
 4. $(a + b)^3 =$
 $(a + b)^2 = a^2 + 2ab + b^2$ $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

For each polynomial:

- state the total number of complex roots
- state the possible number of real roots

Since imaginary numbers always come in pairs the number of real zeros alwavs decreases by 2's

1.
$$f(x) = 7x^3 + 11x^2 - 2x + 13$$

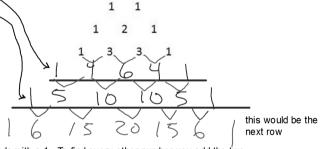
Total # complex roots = 3

2.
$$y = -9x^6 + x^5 - 3x^4 - 21x^3 + x^2 - 16x + 4$$

3.
$$f(x) = 5x^9 - 14x^7 - 4x^2 + 74x - 103$$

Find the next two rows of this pattern. of numbers.

This pattern is called: Pascal's Triangle.



1

Every row starts and ends with a 1. To find every other number you add the two numbers above that spot.

The Row Number in Pascal's Triangle is

the 2nd Number in

each row.

Do you notice a pattern in the exponents?

$$(a+b)^0=1$$
 Powers of a decrease from left to right Powers of b decrease from right to left $(a+b)^1=a+b$ For each term the powers of a and b add to the exponent of $(a+b)^2=a^2+2ab+b^2$
$$(a+b)^3=a^3+3a^2b+3ab^2+b^3$$

$$(a+b)^4=a^3+4a^3b^4+6a^2b^2+4ab^3+b^4$$

Do you see a connection to Pascal's Triangle and the results of expanding powers of (a + b) $\,$?

$$(a+b)^0 = 1$$
 \longrightarrow 1
 $(a+b)^1 = a+b$ \longrightarrow 1 $a+1b$
 $(a+b)^2 = a^2 + 2ab + b^2$ \longrightarrow 1 $a^2 + 2ab + 1b^2$
 $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \longrightarrow$ 1 $a^3 + 3a^2b + 3ab^2 + 1b^3$

The coefficients of each term are the numbers in the row of Pascal's Triangle that to corresponds to the exponent of (a + b).

What do you notice about how many terms each has?

$$(a+b)^{0} = 1$$

$$(a+b)^{1} = a+b$$

$$(a+b)^{2} = a^{2} + 2ab + b^{2}$$

$$(a+b)^{3} = a^{3} + 3a^{2}b + 3ab^{2} + b^{3}$$

$$(a+b)^{4} = a^{4} + 4a^{3}b + 6a^{2}b^{2} + 4ab^{3} + b^{4}$$

The number of terms is one more then the power on (a + b).

Expand this: $(a + b)^5$

If
$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

How would this expansion be different? $(a - b)^4$

$$(a - b)^4 = (a + -b)^4 = a^4 + 4a^3(-b) + 6a^2(-b)^2 + 4a(-b)^3 + (-b)^4$$

= $a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

Do you notice a pattern with the signs?

$$(a-b)^6 = a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$$

$$(a-b)^7 = a^7 - 7a^6b + 21a^5b^2 - 35a^4b^3 + 35a^3b^4 - 21a^2b^5 + 7ab^6 - b^7$$

when there is a minus in the middle of the binomial, the signs of the terms alternate. If a is positive then the first term is positive and the second term becomes negative.

How would the results of these two be different?

$$(a - b)^5$$

This would start with

The second term would be negative and alternate from there.

$$(-a + b)^5$$

This would start with

The second term would be positive and alternate from there.

$$(a - b)^4 = (a + -b)^4 = a^4 + 4a^3(-b) + 6a^2(-b)$$

How would the results of these two be different?

 $(a - b)^4$ $(-a + b)^4$

This would start with a^4 With a^4 because $(-a)^4$ is still positive.

The second term would be negative and alternate from there.

The second term would be negative and alternate from there.