If a polynomial doesn't have rational roots what kind of roots could it have?

1. Irrational Roots
2. Imaginary Roots

What are irrational roots?

Roots that have square roots of non-perfect squares. Example: $x = \sqrt{3}$

Where do irrational roots come from?
Taking square roots.

This happens when.....
1. Using the Quadratic Formula
2. Finding zeros of factors such as $(x^2 - 7)$

Irrational roots always come in PAIRS.

Numbers of the form $11 - \sqrt{5}$ and $11 + \sqrt{5}$ are called CONJUGATES.

Theorem **Irrational Root Theorem**

Let a and b be rational numbers and let $\sqrt{b}$ be an irrational number. If $a + \sqrt{b}$ is a root of a polynomial equation with rational coefficients, then the conjugate $a - \sqrt{b}$ also is a root.

Use the given root to find the remaining 3 roots of this polynomial:

$$f(x) = x^4 + x^3 - 7x^2 - 5x + 10$$

One of the roots is $\sqrt{5}$, $-\sqrt{5}$ The conjugate is one more root

$(x^2-5) \longrightarrow x^2+0x-5$

$$\begin{array}{r} x^2 + x - 2 \\ x^2+0x-5 \overline{) x^4 + x^3 - 7x^2 - 5x + 10} \\ -\ \ x^4 + 0x^3 - 5x^2 \\ \hline +x^3 - 2x^2 - 5x \\ -\ \ x^3 + 0x^2 - 5x \\ \hline -2x^2 + 0x + 10 \\ -\ \ -2x^2 + 0x + 10 \\ \hline 0 \end{array}$$

$x^2 + x - 2$

-2, -1, $+2$, $+1$

$(x-1)(x+2)$

$x = 1, -2$

The other roots are $-2, 1$

Since Imaginary Roots also occur when you are taking a square root, they also come in PAIRS!

These pairs of imaginary roots are called Complex Conjugates.

In other words, if 3 - 5i is a root, so is

3 + 5i.

Theorem **Imaginary Root Theorem**

If the imaginary number $a + bi$ is a root of a polynomial equation with real coefficients, then the conjugate $a - bi$ also is a root.

Use the given roots to find the remaining roots of this polynomial:

$$f(x) = x^5 + 4x^4 + 2x^3 + 76x^2 - 8x + 240$$

Three of the roots are $-6, 2i, 1 + 3i$

their conjugates are also factors

The other two roots are: $\boxed{-2i, 1-3i}$

Use the given root to find the remaining 3 roots of this polynomial: $f(x) = x^4 - 3x^3 + 5x^2 - 27x - 36$

One of the roots is $-3i$

IT's conjugate must also be a root

The other three roots are: $+3i$

$\pm 3i$ came from $\rightarrow x^2+9$

$$\begin{array}{r|l} & x^2 - 3x - 4 \\ x^2+0x+9 & x^4 - 3x^3 + 5x^2 - 27x - 36 \\ & -\ (x^4 + 0x^3 + 9x^2) \\ & -3x^3 - 4x^2 - 27x \\ & -\ (-3x^3 + 0x^2 - 27x) \\ & -4x^2 + 0x - 36 \\ & -\ (-4x^2 + 0x - 36) \\ & 0 \end{array}$$

$\underbrace{x^2-3x-4}$

$(x-4)(x+1)$

4 & -1 are the other factors

Use the given root to find the remaining 2 roots of this polynomial: $y = x^3 - 5x^2 + 11x - 15$

one root is $1 - 2i$

IT's conjugate is also a root

The other two roots are: $\boxed{1+2i, \ 3}$

possible rational roots are

$\pm 1 \ \pm 3 \ \pm 5 \ \pm 15$

$f(1) = -8$

$f(-1) = -32$

$f(3) = 0$

State the other root of the cubic that has the given roots.

Two of the roots are $\sqrt{5}$ and -7

$-\sqrt{5}$ is also a zero

$(x+7)(x^2-5)$

State the equation of this cubic Standard Form.

	x	$+7$
x^2	x^3	$+7x^2$
-5	$-5x$	-35

$\rightarrow \boxed{f(x) = x^3 + 7x^2 - 5x - 35}$

State the other root of the cubic that has the given roots.

two of the roots are -1 and $2+\sqrt{6}$ $\longrightarrow$ $2-\sqrt{6}$ is also a root.

$(x-(2+\sqrt{6}))(x-(2-\sqrt{6}))$

$= (x-2-\sqrt{6})(x-2+\sqrt{6})$

$(x+1)(x^2-4x-2)$

State the equation of this cubic Standard Form.

	x^2	$-4x$	-2
x	x^3	$-4x^2$	$-2x$
$+1$	$+x^2$	$-4x$	-2

	x	-2	$-\sqrt{6}$
x	x^2	$-2x$	$\cancel{-x\sqrt{6}}$
-2	$-2x$	$+4$	$\cancel{+2\sqrt{6}}$
$+\sqrt{6}$	$\cancel{+x\sqrt{6}}$	$\cancel{-2\sqrt{6}}$	-6

$$\boxed{f(x)=x^3-3x^2-6x-2}$$

You can now finish Hwk #33.

Practice Sheet Sec 6-5