If a polynomial doesn't have rational roots what kind of roots could it have?

1. Irrational Roots

2. Imaginary Roots

What are irrational roots?

Roots that have square roots of non-perfect squares. Example: $x = \sqrt{3}$

Where do irrational roots come from?
Taking square roots.

This happens when.....
1. Using the Quadratic Formula
2. Finding zeros of factors such as $(x^2 - 7)$

How many answers do you get when you take the square root of something?

2 $\longrightarrow$ ±

Therefore, if you know there is one irrational root, for example,

if $\sqrt{3}$ is a root, what else is a root? $-\sqrt{3}$

If $11 - \sqrt{5}$ is a root, what else is a root?

$11 + \sqrt{5}$

Irrational roots always come in PAIRS.

Numbers of the form $11 - \sqrt{5}$ and $11 + \sqrt{5}$ are called CONJUGATES.

Theorem	**Irrational Root Theorem**
Let a and b be rational numbers and let $\sqrt{b}$ be an irrational number. If $a + \sqrt{b}$ is a root of a polynomial equation with rational coefficients, then the conjugate $a - \sqrt{b}$ also is a root.	

Use the given roots to find the remaining roots of this polynomial:

$$x^5 - 5x^4 - 2x^3 + 34x^2 - 35x + 7$$

three roots are: $1, \sqrt{7}, 2 - \sqrt{3}$

The other two roots are: $-\sqrt{7}, 2 + \sqrt{3}$

Use the given root to find the remaining 3 roots of this polynomial: $y = x^4 + x^3 - 4x^2 - 2x + 4$

one root is $\sqrt{2}$ $\rightarrow$ $\boxed{-\sqrt{2}}$ must also be a zero.

$\pm\sqrt{2}$ come from $x^2 - 2$.

Do DIVISION with this factor.

$$\begin{array}{r} x^2 + x - 2 \\ x^2 + 0x - 2 \overline{) x^4 + x^3 - 4x^2 - 2x + 4} \\ -\ (x^4 + 0x^3 - 2x^2) \\ x^3 - 2x^2 - 2x \\ -\ (x^3 + 0x^2 - 2x) \\ -2x^2 + 0x + 4 \\ -\ (-2x^2 + 0x + 4) \\ 0 \end{array}$$

factor this Quotient $x^2 + x - 2 = (x-1)(x+2)$

-2, -1, $+2$, $+1$

The remaining two zeros are -2 & 1

Use the given root to find the remaining 2 roots of this polynomial: $y = x^3 - x^2 - 11x + 3$

one root is $2 + \sqrt{3}$ $\boxed{2 - \sqrt{3}}$ must also be a zero.

The 3rd zero must be rational.

possible zeros are $\dfrac{\pm 1, \pm 3}{\pm 1} = \pm 1, \pm 3$

X $f(1) = -8$

X $f(-1) = 12$

$f(3) = -12$ X

$f(-3) = 0$ ✓

The 3rd zero is -3

Find the equation of the cubic that has the given roots.
Give your answer in Standard Form.

two of the roots are 4 and $\sqrt{5}$

If $\sqrt{5}$ is a zero so is $-\sqrt{5}$.

These zeros came from the factor $x^2 - 5$

$(x - 4)(x^2 - 5)$

	x	-4
x^2	x^3	$-4x^2$
-5	$-5x$	$+20$

$x^3 - 4x^2 - 5x + 20$

The original function was $x^3 - 4x^2 - 5x + 20$

What is also true about Imaginary Roots?

They come in PAIRS called Complex Conjugates.

In other words, if 7 + 2i is a root, so is

7 - 2i.

Theorem	Imaginary Root Theorem

If the imaginary number $a + bi$ is a root of a polynomial equation with real coefficients, then the conjugate $a - bi$ also is a root.

Use the given roots to find the remaining roots of this polynomial:

$$x^5 - 8x^4 + 30x^3 - 92x^2 + 189x - 180$$

three of the roots are 4 , $3i$, and $2 - i$

The other two roots are: $-3i$, $2+i$

imaginary zeros always come in pairs

Use the given root to find the remaining 3 roots of this polynomial:

$$y = x^4 + x^3 + 14x^2 + 16x - 32$$

one root is $4i$ $\longrightarrow$ $\boxed{-4i \text{ must also be a zero}}$

If $4i$ & $-4i$ are zeros x^2+16 must be a factor. Use this factor to divide

$$\begin{array}{r} x^2 + x - 2 \\ x^2+0x+16 \overline{\smash{)}\, x^4 + x^3 + 14x^2 + 16x - 32} \\ -\ (x^4 + 0x^3 + 16x^2) \\ \hline x^3 - 2x^2 + 16x \\ -\ (x^3 + 0x^2 + 16x) \\ \hline -2x^2 + 0x - 32 \\ -\ (-2x^2 + 0x - 32) \\ \hline 0 \end{array}$$

$x^2 + x - 2$

-2, 2, -1, 1

$= (x+2)(x-1)$

the other two zeros are -2 & 1