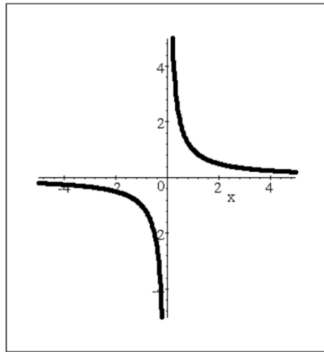


The graph might come down to the HA at the end or it might rise up to the HA at the end



This is the graph of the Parent Reciprocal Function:

$y = 1/x$ the Horizontal Asymptote is $y = 0$

The left-end approaches the HA from BELOW

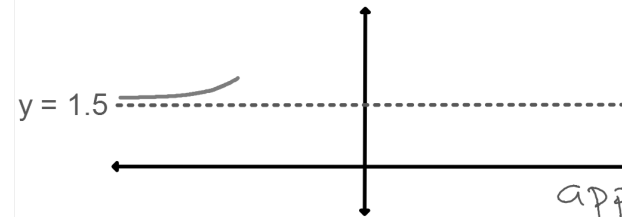
The right-end approaches the HA from ABOVE

$$y = \frac{3x^2 - 4x}{13 + 2x^2}$$

Does the left-end approach the HA from above or below?

Left-end

x	y
-10	1.5962
-100	1.519
-1000	1.502
-10000	1.5002



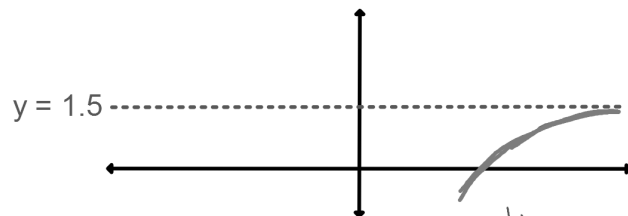
This graph is approaching the HA $y = 1.5$ from above on the Left end

$$y = \frac{3x^2 - 4x}{13 + 2x^2}$$

Does the right-end approach the HA from above or below?

Right-end

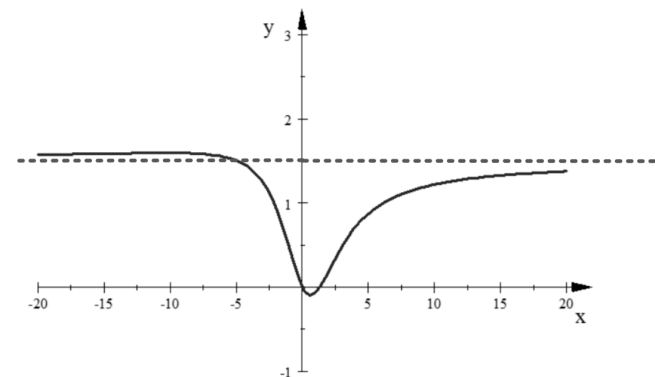
x	y
10	1.2207
100	1.479
1000	1.498
10000	1.4998



This graph is approaching the HA $y = 1.5$ from BELOW on the Right end

This is the actual graph of the function:

$$y = \frac{3x^2 - 4x}{13 + 2x^2}$$



Let's put this all together.

Graph the following Rational Function showing:

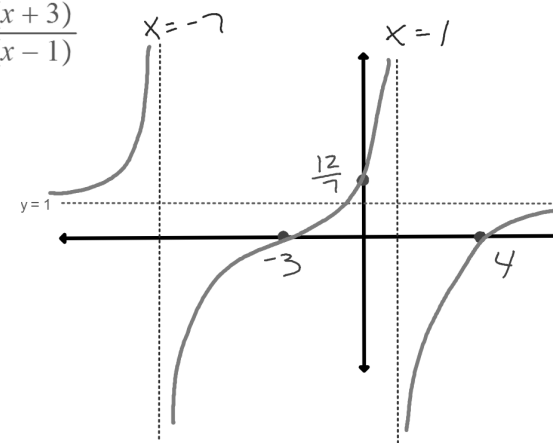
- All asymptotes as dashed lines
- X & Y-intercepts, if any
- Correct behavior around each asymptote.

$$y = \frac{x^2 - x - 12}{x^2 + 6x - 7} = \frac{(x-4)(x+3)}{(x+7)(x-1)}$$

x-int: $4, -3$ y-int: $\frac{12}{7}$

VA: $x = -7, 1$

HA: $y = 1$



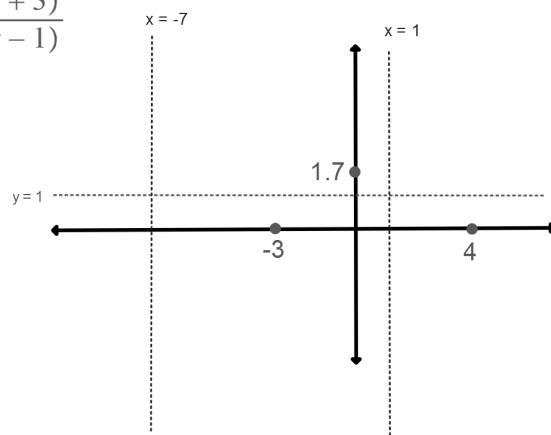
$$y = \frac{x^2 - x - 12}{x^2 + 6x - 7} = \frac{(x-4)(x+3)}{(x+7)(x-1)}$$

Use Sign Analysis to confirm the behavior on the right side of the Vertical Asymptote $x=1$.

When you approach a Vertical Asymptote the value of y gets big positive (up) or big negative (down). To determine which behavior we are getting we need only to test a point barely to the right of 1 (ex: 1.1) and determine if the result is Pos or Neg. It is easiest to use the factor form for this.

X	Y	$\frac{(x-4)(x+3)}{(x+7)(x-1)}$
1.1		$\frac{(-)(+)}{(+)(+)}$

this results in a negative answer which confirms what we originally thought, the graph is going down on the right side of $x=1$.

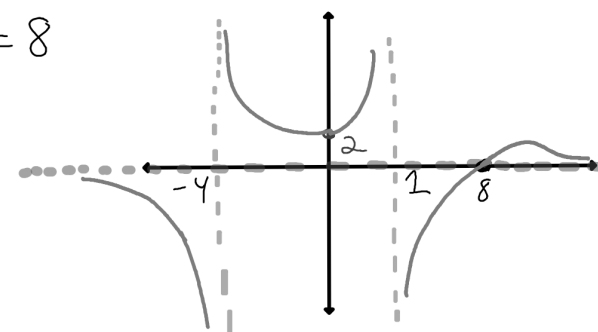


$$y = \frac{x-8}{x^2 + 3x - 4} = \frac{x-8}{(x+4)(x-1)}$$

y-int: $= 2$ x-int: $= 8$

VA: $x = -4, 1$

HA: $y = 0$

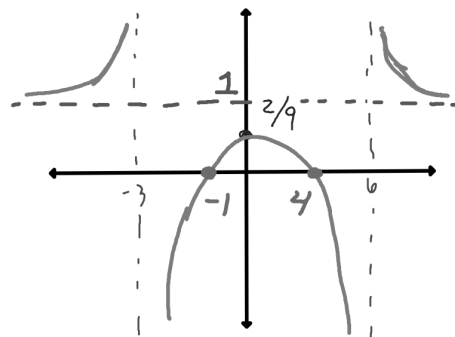


$$y = \frac{(x+1)(x-4)}{(x-6)(x+3)} = \frac{x^2 - 3x - 4}{x^2 - 3x - 18}$$

y-int: $\frac{2}{9}$ x-int: $x = -1, 4$

VA: $x = 6, -3$

HA: $y = 1$

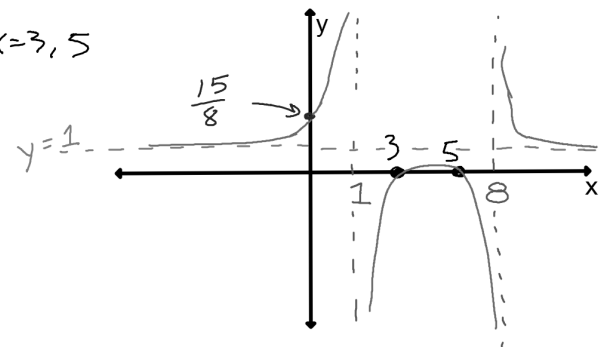


$$y = \frac{(x-3)(x-5)}{(x-1)(x-8)} = \frac{x^2 - 8x + 15}{x^2 - 9x + 8}$$

y-int: $\frac{15}{8}$ x-int: $x = 3, 5$

VA: $x = 1, 8$

HA: $y = 1$



You can now finish Hwk #39

Practice Sheet Graphs of Rational Functions