How could you use Synthetic Division to find the following quotient?

$$\frac{x^4+4x^3-35x^2-78x+360}{x^2+x-20} = x^2+3x-18$$

$(x+5)(x-4)$

-20
5 -4
+1

-5	1	4	-35	-78	360
		-5	5	150	-360
4	1	-1	-30	72	0
		4	12	-72	
	1	3	-18	0	

How could you use Synthetic Division to find the following quotient?

$$\frac{2x^3-17x^2+23x-3}{2x-3} \cdot \frac{\frac{1}{2}}{\frac{1}{2}}$$

Turn the divisor into a linear factor with leading coefficient of 1.

$$\frac{x^3-\frac{17}{2}x^2+\frac{23}{2}x-\frac{3}{2}}{x-\frac{3}{2}}$$

No do synthetic division

$\frac{3}{2}$	1	$-\frac{17}{2}$	$\frac{23}{2}$	$-\frac{3}{2}$
		$\frac{3}{2}$	$-\frac{21}{2}$	$\frac{3}{2}$
	1	-7	1	0

$= x^2-7x+1$

Using the "Box" for Polynomial Division

Expand using the Box:

$(x+3)(2x-5)$

	$2x$	-5
x	$2x^2$	$-5x$
$+3$	$+6x$	-15

$2x^2+x-15$

$x+4 \overline{\smash{)}\, x^2+9x+20}$

$(x+4)(\quad)=x^2+9x+20$

	x	$+5$	
x	x^2	$+5x$	0
$+4$	$4x$	20	

these have to add up to $+9x$

these have to add up to 20

0 is the remainder

Find this quotient using using "the Box"

$$\frac{x^2+8x-13}{x-3}$$

$x+11 \quad R=20$

x	x^2	$11x$	$+20$
-3	$-3x$	-33	

These two spaces have to add to $+8x$

These two spaces have to add to -13

Find this quotient using Polynomial Long Division or using "the Box"

$$\begin{array}{r} 4x^2 + x - 5 \\ 3x - 2 \overline{\big) 12x^3 - 5x^2 - 17x + 8} \\ -\ 12x^3 - 8x^2 \\ \hline 3x^2 - 17x \\ -\ 3x^2 - 2x \\ \hline -15x + 8 \\ -\ -15x + 10 \\ \hline -2 \end{array}$$

$4x^2 + x - 5 \quad R = -2$

$4x^2 + x - 5 \quad R = -2$

	$4x^2$	$+x$	-5	$R=-2$
$3x$	$12x^3$	$3x^2$	$-15x$	-2
-2	$-8x^2$	$-2x$	$+10$	

These two spaces have to add to $-5x^2$

These two spaces have to add to $-17x$

These two spaces have to add to $+8$

Find this quotient: $\dfrac{4x^3 + 7x - 9}{x + 3}$

$-3)$	4	0	7	-9
		-12	36	-129
	4	-12	43	-138

$4x^2 - 12x + 43 \quad R = -138$

OR

$R = -138$

	$4x^2$	$-12x$	$+43$	
x	$4x^3$	$-12x^2$	$43x$	-138
$+3$	$12x^2$	$-36x$	129	

Sec 6-5: Theorems About Roots of Polynomial Equations.

Theorem	Rational Root Theorem

If $\frac{p}{q}$ is in simplest form and is a rational root of the polynomial equation $a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0 = 0$ with integer coefficients, then p must be a factor of a_0 and q must be a factor of a_n.

Theorem	Rational Root Theorem

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What this says is that if a polynomial has any rational roots then they will be amongst the list of roots found by doing the following:

$$\frac{\text{Factors of the constant}}{\text{Factors of the leading coefficient}}$$

State the possible rational roots of this polynomial then find all roots. $x^3 + 2x^2 - 5x - 6$

$$\frac{\pm 1 \quad \pm 2 \quad \pm 3 \quad \pm 6}{\pm 1}$$

$$= \pm 1, \pm 2, \pm 3, \pm 6$$

Since f(-1)=0, -1 is a zero. Now I can do division to find the other zeros:

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array} \rightarrow x^2 + x - 6$$

$$\begin{array}{ccc} & -6 & \\ -2 & & 3 \\ & 1 & \end{array} \longrightarrow (x-2)(x+3)$$

Since x-2 and x+3 are factors the other two zeros are: 2 and -3

State the possible rational roots of this polynomial then find all roots. $2x^3 + 7x^2 + 2x - 3$

$$\frac{\pm 1, \pm 3}{\pm 1, \pm 2}$$

$$\pm 1, \pm\tfrac{1}{2}, \pm 3, \pm\tfrac{3}{2}$$

Since f(-1)=0, -1 is a root. Now I can use division on the original polynomial:

$$\begin{array}{r|rrrr} -1 & 2 & 7 & 2 & -3 \\ & & -2 & -5 & 3 \\ \hline & 2 & 5 & -3 & 0 \end{array} \rightarrow 2x^2 + 5x - 3$$

$$\begin{array}{ccc} & -6 & \\ 6 & & -1 \\ & 5 & \end{array} \longrightarrow \begin{array}{c|c|c} & x & +3 \\ \hline 2x & 2x^2 & 6x \\ \hline -1 & -x & -3 \end{array}$$

Since x+3 and 2x-1 are factors the other two roots are: -3 & $\frac{1}{2}$

You can now finish Hwk #32 Sec 6-5

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Problems 1-5, 32-34