Horizontal Asymptotes:

The horizontal line that a graph approaches as you move farther and farther to the left and right. (END BEHAVIOR)

The value of y that the function approaches as x gets larger and larger (pos and neg).

The function should approach the same value of y on both ends or the asymptote wouldn't be horizontal.

One way to find the horizontal asymptote of a rational function is to use the table function on the graphing calculator.

Enter this equation into Y_1 then go to the table.	$y = \dfrac{4x^2 + 3x - 7}{7x - 2x^2 + 5}$

To find the Horizontal Asymptote, if there is one, enter the following:

Big positive x-values to represent the far right....what is y getting close to?

Big negative x-values to represent the far left....what is y getting close to?

What is the Horizontal Asymptote? $y = -2$

Use this technique to find the Horizontal Asymptote, <u>if there is one</u>, for each of the following:

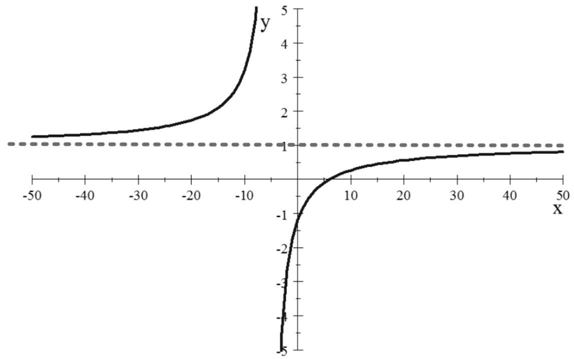
1. $y = \dfrac{x^2 + 9x - 11}{2x^3 - x}$

$y = 0$

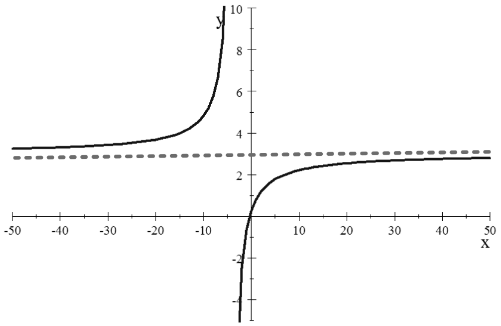
2. $y = \dfrac{5x^2 + 3x - 1}{4x + 7}$

NO HA

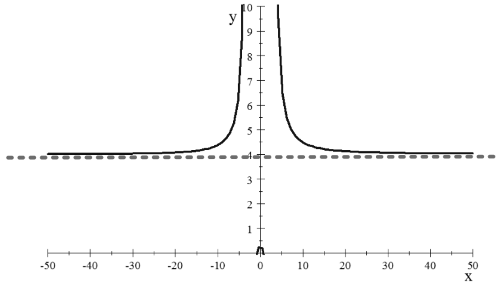
1. $y = \dfrac{x - 6}{x + 5}$ HA: $y = 1$



2. $y = \dfrac{3x+1}{x+4}$ HA: $y = 3$



3. $y = \dfrac{8x^2+x-6}{2x^2-21}$ HA: $y = 4$



What do you notice in the equations that would give you the HA?

1. $y = \dfrac{x-6}{x+5}$ HA: y = 1

2. $y = \dfrac{3x+1}{x+4}$ HA: y = 3

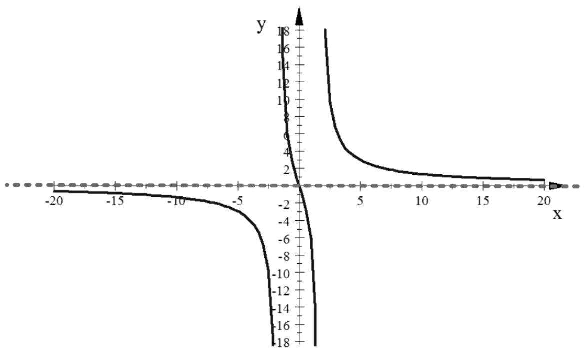
3. $y = \dfrac{8x^2+x-6}{2x^2-21}$ HA: y = 4

What do these three equations have in common?

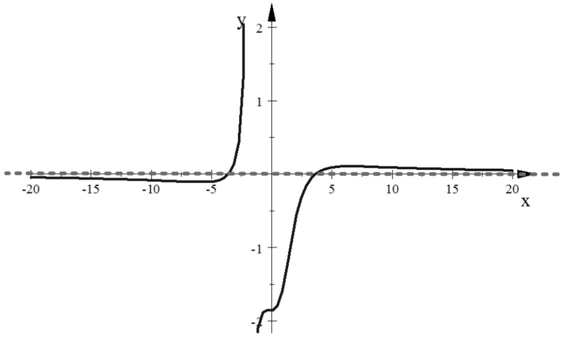
The degree of the numerator and denominator are the same.

When this happens the HA is found by taking the ratio of the leading coefficients.

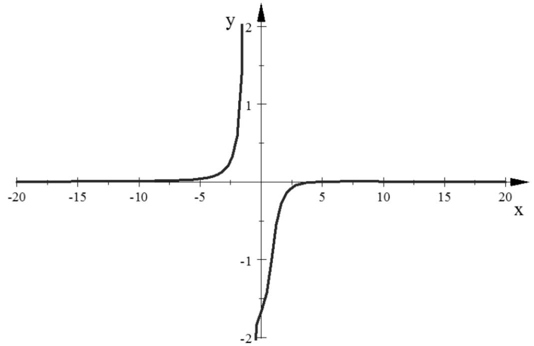
4. $y = \dfrac{4x+9x}{x^2-3}$ HA: $y = 0$



5. $y = \frac{x^2 - 13}{x^3 + 7}$ HA: $y = 0$



6. $y = \frac{x - 5}{2x^3 + 3}$ HA: $y = 0$



What do you notice in the equations that would tell you the HA is y = 0?

4. $y = \frac{4x + 9x}{x^2 - 3}$ HA: y=0

5. $y = \frac{x^2 - 13}{x^3 + 7}$ HA: y=0

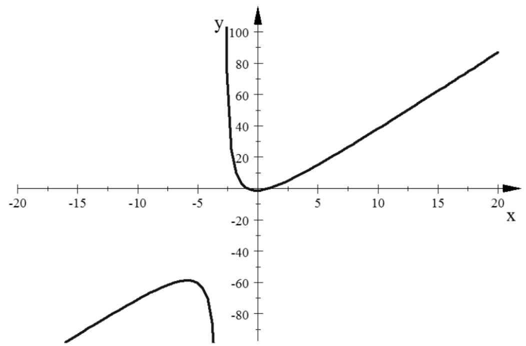
6. $y = \frac{x - 5}{2x^3 + 3}$ HA: y=0

What do these three equations have in common?

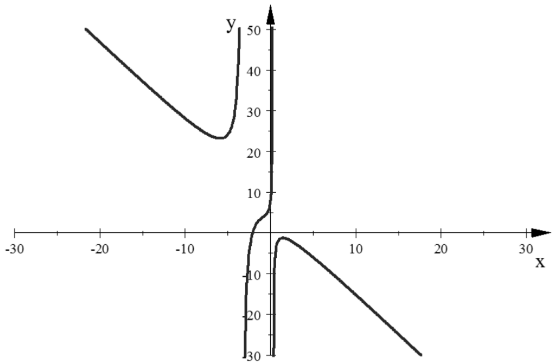
The degree of the denominator is greater than the degree of the numerator.

When this happens the HA is always y = 0

7. $y = \frac{5x^2 - 4}{x + 3}$ HA: NO HA



8. $y = \frac{-2x^3 + 5x - 8}{x^2 + 3x - 1}$ HA: NO HA



What do you notice in the equations that would tell you that there is no HA?

7. $y = \frac{5x^2 - 4}{x + 3}$ HA:

8. $y = \frac{-2x^3 + 5x - 8}{x^2 + 3x - 1}$ HA:

What do these three equations have in common?

The degree of the numerator is greater than the degree of the denominator.

When this happens there is NO HA.

Without using a table or a graph how could you tell from the equation what the horizontal asymptote is or if it even has one?

Predict the Horizontal Asymptote for each of the rational functions below, if any.

a. $y = \frac{10x + 7}{5x - 3}$

HA: $y = \frac{10}{5} = 2$

b. $y = \frac{6x^2 - 5}{2x + 3}$

HA: NO HA

c. $y = \frac{12x - 11}{3x^2 - 1}$

HA: $y = 0$

Horizontal Asymptotes: Depends on the degrees of the Numerator and the Denominator

Case 1: Degree of the Numerator > Degree of the Denominator

No HA

Case 2: Degree of the Numerator = Degree of the Denominator

HA: y = ratio of the Leading Coefficients

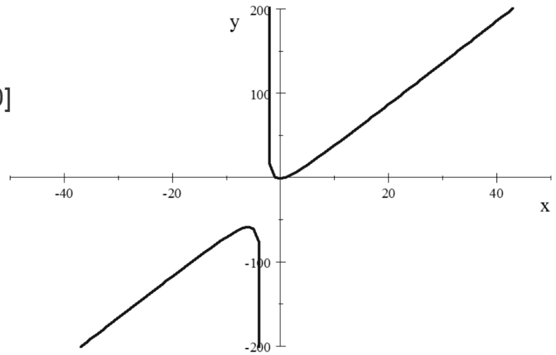
Case 1: Degree of the Denominator > Degree of the Numerator

HA: y = 0

7. $y = \frac{5x^2 - 4}{x + 3}$ HA:

This graph has what is called a Slant Asymptote.

window
x [-20,10]
y [-100,50]



Find this quotient $\frac{5x^2 - 4}{x + 3}$

$$\begin{array}{r|rrr} -3 & 5 & 0 & -4 \\ & & -15 & 45 \\ \hline & 5 & -15 & 41 \end{array}$$

$= 5x - 15 \quad R = 41$

Enter the quotient, without the remainder, in Y_2 and graph.

The equation for the end-behavior asymptote of a Rational Function is:

The quotient without the remainder

This rational function has the following slant asymptote:

y = 5x - 15