When the denominator of a rational function is zero the function is undefined.

Because this value of x can never be used this leads to a break in the graph (it's not continuous)

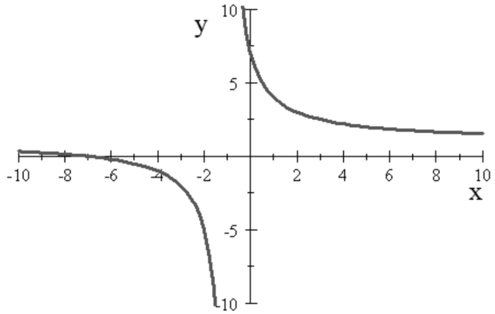
These breaks in the graph are one of two types:

Vertical Asymptotes

Holes

Graph the rational function f(x) in a standard window.

$$f(x) = \frac{x+7}{x+1}$$



There is a break in the graph at x = -1

This kind of break in the graph is called a

Vertical Asymptote

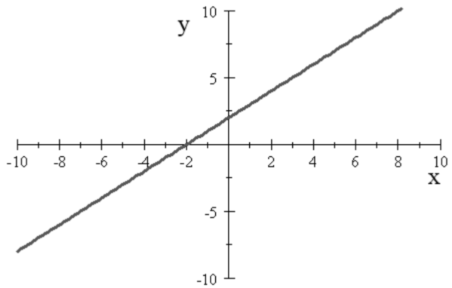
Graph the rational function f(x) in a standard window.

$$f(x) = \frac{((x-3)(x+2))}{(x-3)}$$

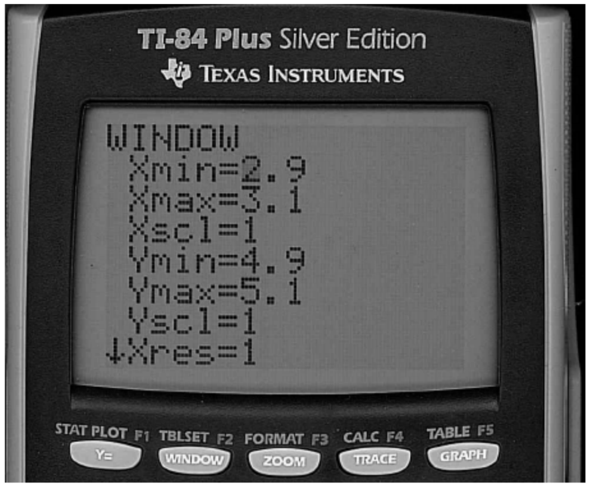
Do you see a vertical asymptote?

Why do you think that there isn't a vertical asymptote at x = 3?

Because the factor x-3 cancels and it's <u>almost</u> like It wasn't there.



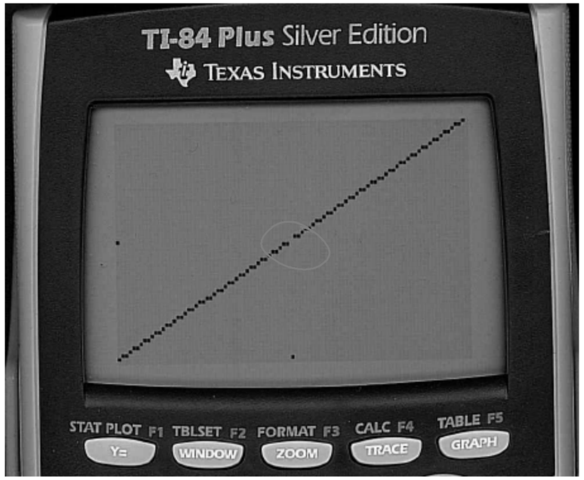
Change the window to the following:



What do you see?

This kind of break in the graph is called a **Hole**

$$f(x) = \frac{((x-3)(x+2))}{(x-3)}$$



Why did this graph have a Vertical Asymptote at x = -1

$$f(x) = \frac{x+7}{x+1}$$

X+1 occurs in ONLY the denominator

but

this graph had a hole at x = 3?

$$f(x) = \frac{(\cancel{x-3})(x+2)}{(\cancel{x-3})}$$

Because (X-3) is a factor found in BOTH the numerator & the denominator

Breaks in the graph are caused by zeros of the denominator and are called:

Points of Discontinuity

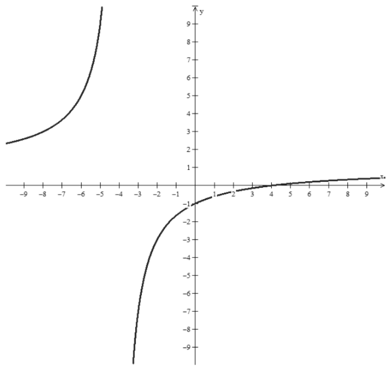
Holes or **Vertical Asymptotes**

Occur at values of x that are zeros of both the denominator AND numerator

Occur at values of x that are zeros of the denominator ONLY.

An exception to this rule:

$$y = \frac{x^2-16}{x^2+8x+16} = \frac{(x+4)(x-4)}{(x+4)(x+4)}$$



Why is there a VA at x = -4 and not a hole?

Even though the factors (x+4) are common to the numerator and denominator, when you cancel them there is still (x+4) left in the denominator.

Properties | **Vertical Asymptotes**

The rational function $f(x) = \frac{P(x)}{Q(x)}$ has a point of discontinuity for each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have no common real zeros, then the graph of $f(x)$ has a vertical asymptote at each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have a common real zero a, then there is a hole in the graph or a vertical asymptote at $x = a$.

Find any points of dicontinuity and classify them as Vertical Asymptotes or Holes.

1. $y = \dfrac{2x(x+1)(x-6)}{(x-1)(x-6)}$

Pts of Discontinuity: $x = 1, 6$

VA: $x = 1$ → 1 is only a zero of the denominator

Holes: $x = 6$ → 6 is a zero of both the numerator & the denominator

2. $y = \dfrac{3x^2-6}{x^2-4} = \dfrac{3(x^2-2)}{(x+2)(x-2)}$

Pts of Discontinuity: $x = \pm 2$

VA: $x = -2, 2$

Holes: NONE

3. $y = \dfrac{x^2-x-12}{x^2-16} = \dfrac{(x-4)(x+3)}{(x+4)(x-4)}$

-12

-4 $+3$

-1

Pts of Discontinuity: $x = \pm 4$

VA: $x = -4$

Holes: $x = 4$

4. $y = \frac{x^2 + 6x + 9}{x^2 + 5x + 6} = \frac{(x+3)(x+3)}{(x+3)(x+2)}$

Pts of Discontinuity: $x = -3, -2$

6
3 2
5

9
3 3
6

VA: $x = -2$

Holes: $x = -3$

5. $y = \frac{2x^2}{x^2 + 3}$

Pts of Discontinuity: NONE

VA: NONE

Holes: NONE

There are no points of discontinuity because the denominator has no real zeros!

You can now finish:

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Problems 2, 3, 5, 12, 13, 17, 18,