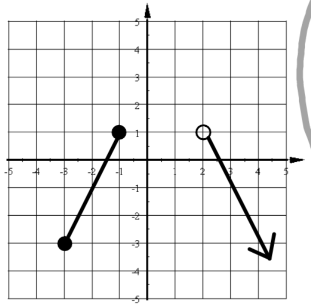
The concept of an Inverse Relation is all about...

switching X and Y

Original Relation		Inverse Relation
$f(x)$	Becomes	$f^{-1}(x)$
The point (a,b)	Becomes	The point (b,a)
Domain of $f(x)$	Becomes	Range of $f^{-1}(x)$
Range of $f(x)$	Becomes	Domain of $f^{-1}(x)$
Graph of $f(x)$	Reflect over y=x Becomes	Graph of $f^{-1}(x)$

Use this graph of f(x):



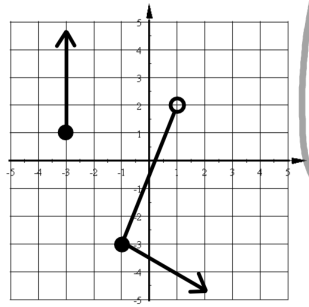
Domain of f(x): $-3 \le x \le -1$, $x > 2$

Range of f(x): $y \le 1$

Domain of $f^{-1}(x)$: $x \le 1$

Range of $f^{-1}(x)$: $-3 \le y \le -1$, $y > 2$

Use this graph of f(x):



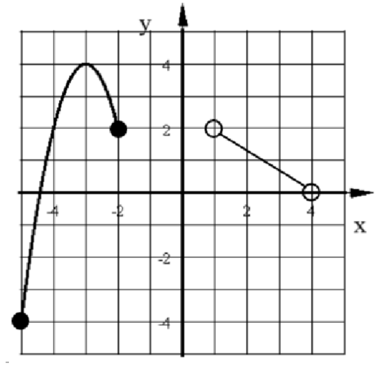
Domain of f(x): $x = -3$, $x \ge -1$

Range of f(x): $\mathbb{R}$

Domain of $f^{-1}(x)$: $\mathbb{R}$

Range of $f^{-1}(x)$: $y = -3$, $y \ge -1$

1. Given the graph of f(x) below state the domain and range of $f^{-1}(x)$



Domain of $f^{-1}(x)$: $-4 \le x \le 4$

this is the range of the original function

Range of $f^{-1}(x)$: $-5 \le y \le -2$

$1 < y < 4$

this is the domain of the original function

What I want you to know from Sec 7-7:

1. Given an original relation be able to tell if the inverse is a function or not.
2. Know the relationship between the Domain and Range of an original relation and the Domain and Range of the inverse relation.
3. Be able to write the equation of the inverse relation.

Solve for K

$$G = T(ZK - Q)^2 - H$$

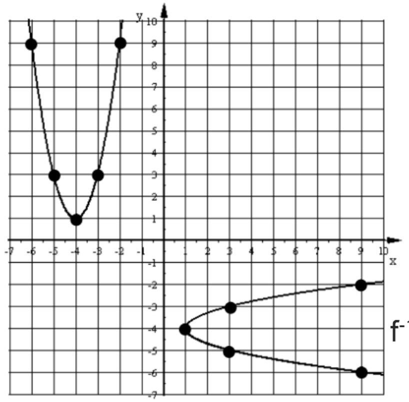
$$K = \frac{\pm\sqrt{\frac{G+H}{T}} + Q}{Z}$$

Solve for A

$$\sqrt{\frac{E - CA}{W}} + R = X$$

$$A = \frac{W(X-R)^2 - E}{-C}$$

$f(x) = 2(x+4)^2 + 1$



Equations of Inverses

1. Switch the variables x and y
2. Solve equation for y

step 1: $x = 2(y+4)^2 + 1$

step 2:

$f^{-1}(x) = \pm\sqrt{\frac{x-1}{2}} - 4$

Find the equation of the inverse for each function and tell if $f^{-1}(x)$ is a function.

1. $f(x) = 2x - 3$

$x = 2y - 3$

$f^{-1}(x) = \frac{x+3}{2} = \frac{1}{2}x + \frac{3}{2}$

2. $f(x) = (x+5)^3 - 7$

$x = (y+5)^3 - 7$

$f^{-1}(x) = \sqrt[3]{x+7} - 5$

Find the equation of the inverse for each function and tell if $f^{-1}(x)$ is a function.

3. $y = 2(x-3)^4 + 7$

$x = 2(y-3)^4 + 7$

$f^{-1}(x) = \pm\sqrt[4]{\frac{x-7}{2}} + 3$

4. $y = \frac{-4}{x+7} - 8$

$x = \frac{-4}{y+7} - 8$

$f^{-1}(x) = \frac{-4}{x+8} - 7$

Find the equation of the inverse.

$$y = \frac{(4x+7)^3 - 2}{3} \rightarrow x = \frac{(4y+7)^3 - 2}{3}$$

$$f^{-1}(x) = \frac{\sqrt[3]{3x+2} - 7}{4}$$

Find the equation of the inverse.

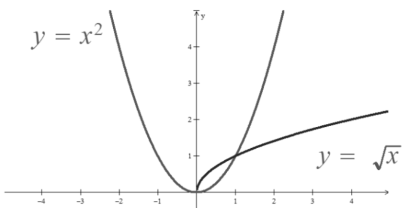
$$y = 2\left(\frac{\sqrt{6x+1}}{11}\right)^5 - 3 \longrightarrow x = 2\left(\frac{\sqrt{6y+1}}{11}\right)^5 - 3$$

$$f^{-1}(x) = \frac{\left(11\sqrt[5]{\frac{x+3}{2}}\right)^2 - 1}{6}$$

Since $\sqrt{\ }$ is the inverse of $(\ )^2$ what would you expect the graph of $\sqrt{\ }$ to look like?

Sideways Parabola ?

Graph both $y = x^2$ and $y = \sqrt{x}$ in a Standard Window.



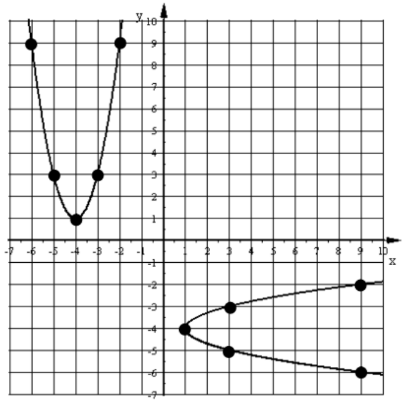
Since $(\ )^2$ and $\sqrt{\ }$ are inverses why is the graph of $\sqrt{\ }$ only half of a sideways parabola.

$$y = \sqrt{x}$$

Why is the graph of the above only "half a sideways parabola"?

- If it were both halves then it wouldn't be a function.
- Without a sign in front of the radical it means the Principal Square Root (positive root).

$f(x) = 2(x+4)^2 + 1$



$f^{-1}(x) = \pm\sqrt{\frac{x-1}{2}} - 4$

Original Function

$f(x) = 2(x+4)^2 + 1$ $\longrightarrow$

Inverse Relation

$f^{-1}(x) = \pm\sqrt{\frac{x-1}{2}} - 4$

Since the inverse of f(x) isn't a function we only use the positive part of the inverse.

Therefore, the graph of a Square Root function will be half of a sideways parabola.

$y = a(x - h)^2 + k$

h: Horizontal Translation

k: Vertical Translation

a: a>1 Vertical Stretch
0<a<1 Vertical Shrink
a is neg: x-axis reflection (upside down)

Vertex:

(h,k)

Describe the transformations to the parent function the following equation represents.

$y = -3(x + 1)^2 - 8$

- x-axis reflection (upside down)
- Vertical Stretch Factor of 3 (3 times taller)
- Shift 1 unit left
- Shift 8 units down

vertex (-1,-8)

$y = 8(x - 9)^2 - 4$

State the Vertex of this parabola.

$(9, -4)$

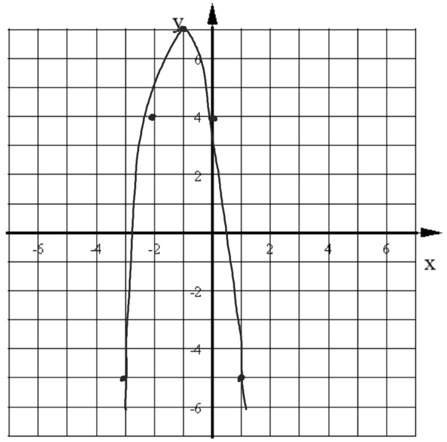
Graph this parabola:

$y = -3(x+1)^2 + 7$

Vertex $(-1, 7)$

3x Taller

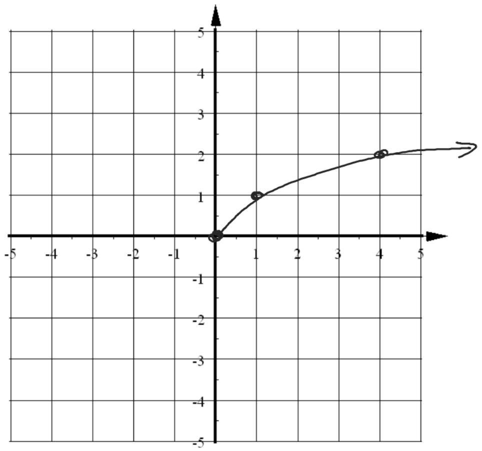
upsidedown

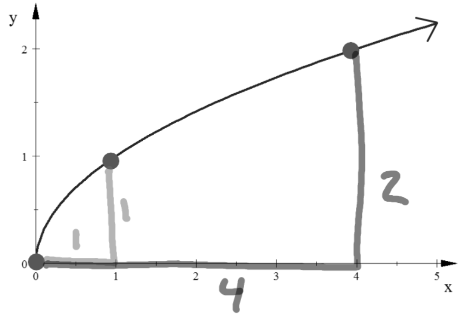


Graph of the Parent Function:

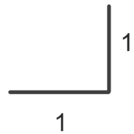
$y = \sqrt{x}$

x	y
0	0
1	1
4	2





First "Good Point"



Second "Good Point"

What do you think $y = \sqrt{x - 3}$ looks like?

The parent function shifted 3 units right

What do you think $y = \sqrt{x} + 7$ looks like?

The parent function shifted 7 units up