Theorem	Remainder Theorem
If a polynomial $P(x)$ of degree $n \geq 1$ is $\boxed{\text{divided by } (x-a)}$, where a is a constant, then the remainder is $P(a)$.	

This can only be applied when the divisor is linear.

Is x + 5 a factor of $2x^3 + x^2 - 4x + 15$?

Only if the remainder is zero!

DIVIDE

$$
\begin{array}{r}
2x^2 - 9x + 44 \\
x+5 \,\overline{)\, 2x^3 + x^2 - 4x + 15} \\
-\ 2x^3 + 10x^2 \\
\hline
-9x^2 - 4x \\
-\ -9x^2 - 45x \\
\hline
44x + 15 \\
-\ 44x + 220 \\
\hline
-205
\end{array}
$$

x+5 is NOT A factor Because the remainder isn't zero

OR

use remainder theorem

$f(-5) = 2(-5)^3 + (-5)^2 - 4(-5) + 15$

$= -250 + 25 + 20 + 15$

$= -205$

This is the remainder and it isn't zero So x+5 is NOT a factor

Is $x - 3$ a factor of $2x^3 - 12x^2 + 21x - 9$?

DIVIDE

$$
\begin{array}{r}
2x^2 - 6x + 3 \\
x-3 \,\overline{)\, 2x^3 - 12x^2 + 21x - 9} \\
-\ 2x^3 - 6x^2 \\
\hline
-6x^2 + 21x \\
-\ -6x^2 + 18x \\
\hline
3x - 9 \\
-\ 3x - 9 \\
\hline
0
\end{array}
$$

OR

use remainder theorem

$f(3)$

$= 2(27) - 12(9) + 21(3) - 9$

$54 - 108 + 63 - 9$

$= 0$

Since the remainder is zero x-3 must be a factor

What is the remainder of this quotient?

$$\frac{6x^2 + 5x - 2}{x - 4} = 6x + 29 + \frac{114}{x-4} \rightarrow R = 114$$

OR

$6(4)^2 + 5(4) - 2$

$96 + 20 - 2$

$116 - 2 = 114$

-1 and 2 are solutions of the equation below. The other two solutions are imaginary.
How would you find the other two?

$(x+1)(x-2)(x^2+4)$

$$x^4 - x^3 + 2x^2 - 4x - 8 = 0$$

Start by dividing by one of the factors. Then divide this quotient by the other factor. After the second division you are left with a quadratic you can use to find the last two zeros.

$x^3 - 2x^2 + 4x - 8$

x+1 $\overline{)\,x^4 - x^3 + 2x^2 - 4x - 8}$

$-\ x^4 + x^3$

$-2x^3 + 2x^2$

$-\ -2x^3 - 2x^2$

$4x^2 - 4x$

$-4x^2 + 4x$

$-8x - 8$

$-\ -8x - 8$

0

$x^2 + 4$

$x-2\ \overline{)\,x^3 - 2x^2 + 4x - 8}$

$-\ x^3 - 2x^2$

$0\ 0\quad 4x - 8$

$-\ 4x - 8$

0

If $x + 2$ is a factor, use polynomial division to factor $x^3 + x^2 - 22x - 40$ completely.

$x^3 + x^2 - 22x - 40 =$

$(x+2)(x^2 - x - 20)$

$x^2 - x - 20$

$x+2\ \overline{)\,x^3 + x^2 - 22x - 40}$

$-\ x^3 + 2x^2$

$-x^2 - 22x$

$-\ -x^2 - 2x$

$-20x - 40$

$-\ -20x - 40$

0

-20

$-5 \quad 4$

-1

$\boxed{(x+2)(x-5)(x+4)}$

Graph to find all REAL solutions then find the remaining imaginary solutions.

$x^3 - 4x^2 + x + 26 = 0$ x = -2

since -2 is a real solution what is a factor of the original polynomial?

(x+2)

If x+2 is a factor use polynomial division to find the other factor.

$x^2 - 6x + 13$

x+2 $\overline{)\,x^3 - 4x^2 + x + 26}$

$-\ x^3 + 2x^2$

$-6x^2 + x$

$-\ -6x^2 - 12x$

$13x + 26$

$-\ 13x + 26$

0

Find the zeros of this other factor.
(use Quadratic Formula)

$x^2 - 6x + 13$

$$\frac{6 \pm \sqrt{-16}}{2}$$

$$\frac{6 \pm 4i}{2}$$

$$\boxed{3 \pm 2i}$$

You can now finish Hwk #30 Sec 6-3

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Due Monday

Problems 34, 35, 37, 40, 42-44