Sec 6-4: Solving Polynomial Equations

- Solve by factoring:
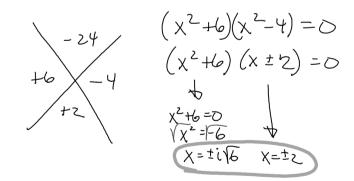 finds both real and imaginary zeros.

- Solve by graphing:
  finds only real zeros

Every polynomial has exactly n solutions, where n is the degree of the polynomial.

Some of these solutions may be imaginary so not all solutions can be found by graphing.

Solve by factoring (same question as if I asked you to find the zeros of the function!)

Find ALL Complex solutions.

this means to find both real and imaginary solutions

$$6x^6 - 12x^4 - 378x^2 = 0$$
$$6x^2(x^4 - 2x^2 - 63) = 0$$

$$\frac{-63}{-9 \quad +7}$$
$$-2$$

$$6x^2(x^2 - 9)(x^2 + 7) = 0$$
$$6x^2(x \pm 3)(x^2 + 7) = 0$$

$$\boxed{X = 0, \pm 3, \pm i\sqrt{7}}$$

$$x^2 + 7 = 0$$
$$\sqrt{x^2} = \sqrt{-7}$$
$$x = \pm i\sqrt{7}$$

Find ALL Complex solutions.
$$x^4 + 2x^2 - 24 = 0$$

$$\frac{-24}{+6 \quad -4}$$
$$+2$$

$$(x^2 + 6)(x^2 - 4) = 0$$
$$(x^2 + 6)(x \pm 2) = 0$$

$$x^2 + 6 = 0$$
$$\sqrt{x^2} = \sqrt{-6}$$
$$\boxed{X = \pm i\sqrt{6} \quad X = \pm 2}$$

Find ALL Complex solutions.

$$x^3 + 3x^2 - 4x - 12 = 0$$

How would you factor a polynomial with four terms?

	X	$+3$
X^2	X^3	$+3x^2$
-4	$-4x$	-12

$$(x+3)(x^2-4) = 0$$
$$(x+3)(x+2)(x-2) = 0$$

$$\boxed{X = -3, -2, 2}$$

Find ALL Complex solutions.

Another technique To factor 4 Terms is called: **FACTOR BY Grouping**

$$\underbrace{2x^3 - 3x^2}_{GCF} + \underbrace{10x - 15}_{GCF} = 0$$

$$x^2(2x-3) + 5(2x-3)$$

$$(2x-3)(x^2+5) = 0$$

$$\boxed{X = \tfrac{3}{2}, \pm i\sqrt{5}}$$

$$x^2 + 5 = 0$$
$$\sqrt{x^2} = \sqrt{-5}$$

Find ALL Complex solutions.

$$2x^5 - 72x = 0$$

$$2x(x^4 - 36)$$

$$2x(x^2-6)(x^2+6)$$

$$x^2 - 6 = 0$$
$$\sqrt{x^2} = \sqrt{6}$$

$$x^2 + 6 = 0$$
$$\sqrt{x^2} = \sqrt{-6}$$

$$\boxed{X = 0, \pm\sqrt{6}, \pm i\sqrt{6}}$$

Find ALL Complex solutions.

$$x^5 + 9x^3 - 36x = 0$$

$$x(x^4 + 9x^2 - 36) = 0$$

$$x(x^2+12)(x^2-3) = 0$$

$$\begin{array}{c} -36 \\ +12 \diagdown -3 \\ +9 \end{array}$$

$$\boxed{X = 0}$$

$$x^2 + 12 = 0$$
$$x^2 = -12$$
$$X = \pm\sqrt{-12} = \boxed{\pm 2i\sqrt{3}}$$

$$x^2 - 3 = 0$$
$$x^2 = 3$$
$$\boxed{X = \pm\sqrt{3}}$$

Find ALL Complex solutions.

$$2x^4 + x^2 - x = 16x^3 - 13x^2 + 111x$$

$$2x^4 - 16x^3 + 14x^2 - 112x = 0$$

NOW START FACTORING

$$x^2 + 7 = 0$$
$$\sqrt{x^2} = \sqrt{7}$$

$$2x(x^3 - 8x^2 + 7x - 56) = 0$$
$$x \quad -8$$

	x^3	$-8x^2$
x^2	x^3	$-8x^2$
$+7$	$+7x$	-56

$\rightarrow 2x(x-8)(x^2+7) = 0$

$\searrow x = 0 \quad \searrow x = 8 \quad \searrow$

$$x^2 + 7 = 0$$
$$\sqrt{x^2} = \sqrt{-7}$$
$$x = \pm i\sqrt{7}$$

$$\boxed{x = 0, 8, \pm i\sqrt{7}}$$