Where are the ends of a graph found?

At the far left and far right

When you are asked to describe the end behavior of a graph you are really asked to describe what the value of Y is doing at the very far left and right of the graph.

At the ends of a graph Y will be doing only one of three things:

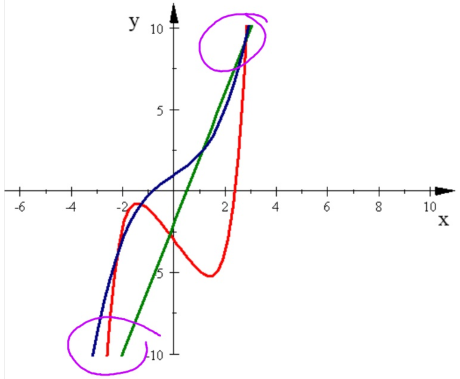
- Increasing
- Decreasing
- remaining Constant

Polynomials

Graph all three of these in a Standard Window:

$$Y_1 = 4x - 2$$
$$Y_2 = 0.25x^3 + x + 1$$
$$Y_3 = 0.1x^5 - 2x - 3$$

What do the graphs have in common? Same end behavior



$$Y_1 = 4x - 2$$
$$Y_2 = 0.25x^3 + x + 1$$
$$Y_3 = 0.1x^5 - 2x - 3$$

What do the equations have in common?

	Degree	Lead Coeff
$Y_1 = 4x - 2$	1	4
$Y_2 = 0.25x^3 + x + 1$	3	.25
$Y_3 = 0.1x^5 - 2x - 3$	5	.1
	ODD	pos

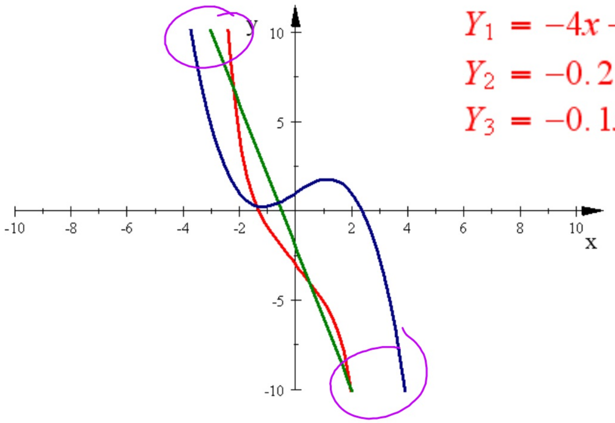
these are all Positive Odd Polynomials

$Y_1 = 4x - 2$

$Y_2 = 0.25x^3 + x + 1$

$Y_3 = 0.1x^5 - 2x - 3$

What would happen if they all had a negative leading coefficient?



$Y_1 = -4x - 2$

$Y_2 = -0.25x^3 + x + 1$

$Y_3 = -0.1x^5 - 2x - 3$

Same end behavior

these are all Negative Odd polynomials

Odd Functions: Largest exponent is ODD when expanded
This is called the degree of the function.

Positive Leading Coefficient:
Moves from the third quadrant to the first quadrant.
Like a line with a Positive slope

Negative Leading Coefficient:
Moves from the second quadrant to the fourth quadrant.
Like a line with a Negative slope

Odd Functions

Positive Leading Coefficient:

Moves from the third quadrant to the first quadrant.

Like a line with a Positive slope

Negative Leading Coefficient:

Moves from the second quadrant to the fourth quadrant.

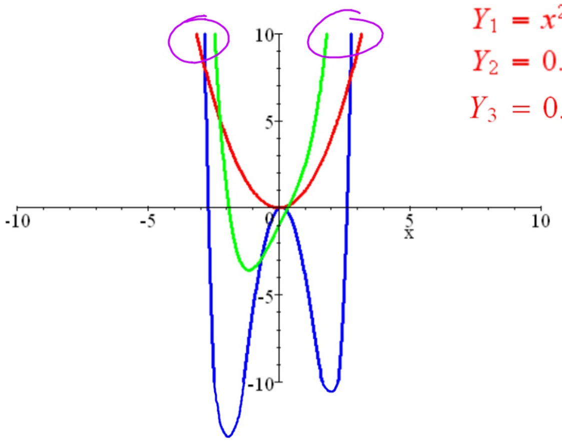
Like a line with a Negative slope

This is called the END BEHAVIOR of an ODD function

Graph all three of these in a Standard Window:

$$Y_1 = x^2$$
$$Y_2 = 0.5x^4 + 3x - 1$$
$$Y_3 = 0.1x^6 - 5x^2 + x$$

What do the graphs have in common?



$$Y_1 = x^2$$
$$Y_2 = 0.5x^4 + 3x - 1$$
$$Y_3 = 0.1x^6 - 5x^2 + x$$

Same end behavior

What do the equations have in common?

	Degree	Lead Coeff
$Y_1 = x^2$	2	1
$Y_2 = 0.5x^4 + 3x - 1$	4	.5
$Y_3 = 0.1x^6 - 5x^2 + x$	6	.1
	even	pos

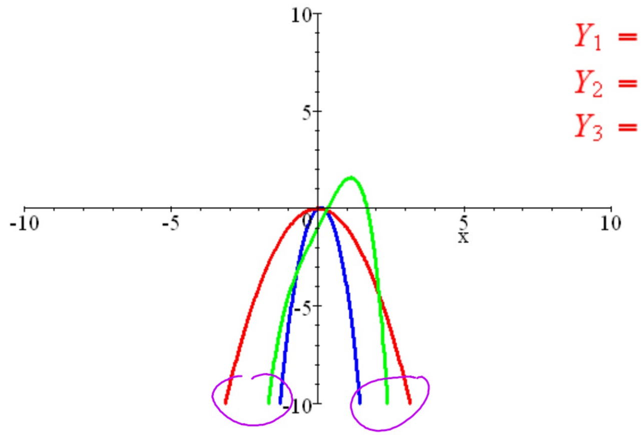
these are all Pos Even polynomials

$Y_1 = x^2$

$Y_2 = 0.5x^4 + 3x - 1$

$Y_3 = 0.1x^6 - 5x^2 + x$

What would happen if they all had a negative leading coefficient?



$Y_1 = -x^2$

$Y_2 = -0.5x^4 + 3x - 1$

$Y_3 = -0.1x^6 - 5x^2 + x$

Same end behavior

Neg evens

Even Functions: Largest exponent is EVEN when expanded

This is called the degree of the function.

Positive Leading Coefficient:

Moves from the second quadrant to the first quadrant.

Like a parabola with a>0

Negative Leading Coefficient:

Moves from the third quadrant to the fourth quadrant.

Like a parabola with a<0

Even Functions

Positive Leading Coefficient:

Moves from the second quadrant to the first quadrant.

Like a parabola with a>0

Negative Leading Coefficient:

Moves from the third quadrant to the fourth quadrant.

Like a parabola with a<0

This is called the END BEHAVIOR of an EVEN function

End-Behavior:

The behavior of the graph on the far left and the far right.

How the value of the function (y) changes as x becomes
larger negative LEFT END $x \to -\infty$
and
larger positive RIGHT END. $x \to \infty$

END BEHAVIOR

EVEN Functions:

Positive Leading Coefficient:	Negative Leading Coefficient:	
(↖ , ↗)	(↙ , ↘)	Our Book
as $x \to -\infty, y \to \infty$	as $x \to -\infty, y \to -\infty$	Other Authors & Mathematicians
as $x \to \infty, y \to \infty$	as $x \to \infty, y \to -\infty$	

END BEHAVIOR

ODD Functions:

Positive Leading Coefficient:	Negative Leading Coefficient:	
(↙ , ↗)	(↖ , ↘)	Our Book
as $x \to -\infty, y \to -\infty$	as $x \to -\infty, y \to \infty$	Other Authors & Mathematicians
as $x \to \infty, y \to \infty$	as $x \to \infty, y \to -\infty$	

State the end behavior of each polynomial.

1. $y = 4x^3 - 6x^2 + 11x - 93$ (↙ , ↗)
   POS ODD
2. $y = 5x(x+2)(x-7)^2$ (↖ , ↗)
   PO EVEN
3. $f(x) = 9x + 6x^2 - x^3 + 13$ (↖ , ↘)
   NEG ODD
4. $y = (9x-7)(4-x)$ NEG EVEN (↙ , ↘)
5. $f(x) = -9x^2(3x-7)^2(8-2x)^3(1-4x)^5$ (↙ , ↘)
   NEG EVEN

You can now finish Hwk #24:

Page 312

Problems 1-10