

When using Real #'s

$(a + b)^2$ is **ALWAYS** a **TRINOMIAL**.

$(x+5)^2$ is never just 2 terms!!!!

$$(x+5)^2 = x^2 + 10x + 25$$

HOWEVER: When using Imaginary #'s
 $(a + bi)^2$ is **ALWAYS** a **BINOMIAL**.

$$\begin{aligned}(1+5i)^2 &= 1^2 + 10i + 25i^2 \\ &= 1 + 10i + 25(-1) \\ &= 1 + 10i - 25 \\ &= -24 + 10i\end{aligned}$$

$(a + bi)^2$ is **ALWAYS**
another Complex Number.

When dealing with Real Numbers only:

$(x + 5)(3x + 2)$ is a Trinomial

When dealing with Complex Numbers:

$(5 + i)(2 + 3i)$ is a Binomial

it's another Complex #

Factors such as $(a + b)$ and $(a - b)$
are called **CONJUGATES**

Conjugate

The conjugate is where we **change the sign in the middle** of two terms like this:

$$\begin{array}{c} 3x + 1 \\ \downarrow \\ \text{Conjugate: } 3x - 1 \end{array}$$

When using REAL #'s:

$$(a + b)(a - b) = a^2 - b^2$$

When using COMPLEX #'s:

$$\begin{aligned}(a + bi)(a - bi) &= a^2 - (bi)^2 \\ &= a^2 - b^2 i^2 \\ &= a^2 - b^2(-1) \\ &= a^2 + b^2\end{aligned}$$

It turns out to be a Constant!

$$i = \sqrt{-1} = i$$

$$i^2 = -1$$

$$i^3 = -i$$

Simplify each.

1. $(3i)(7i)(2i) =$

$$21i^2$$

$$21(-1)$$

$$(-21)(2i)$$

$$\boxed{-42i}$$

2. $(-2i)(10i)(i)(-5) =$

$$-20i^2$$

$$-20(-1)$$

$$(20) \cdot (-5i)$$

$$\boxed{-100i}$$

3. $(2 - 5i)^3 = (2 - 5i)(2 - 5i)(2 - 5i)$

Multiply the first two factors then take that result and multiply by the third factor.

$$\begin{array}{r|rr} 2 & 2 & -5i \\ -5i & 4 & -10i \\ & -10i & 25i^2 \end{array}$$

$$25i^2 - 20i + 4$$

$$(-1)$$

$$-25$$

$$-21 - 20i \rightarrow (-21 - 20i)(2 - 5i)$$

$$\begin{array}{r|rr} 2 & 2 & -5i \\ -21 & -42 & 105i \\ -20i & -40i & 100i^2 \end{array}$$

$$100i^2 + 65i - 42$$

$$(-1)$$

$$-100$$

$$\boxed{-142 + 65i}$$

Find ALL solutions using Square Roots:

$$\begin{aligned}(x+1)^2 + 21 &= 5 \\ -21 \quad -21 \\ \sqrt{(x+1)^2} &= \sqrt{-16} \\ x+1 &= \frac{\pm 4i}{-1} \\ X &= -1 \pm 4i\end{aligned}$$

Now, **all** quadratic equations have 2 solutions.

Some of these solutions may be imaginary.

Find all Complex Solutions (real and imaginary).

$$\begin{aligned}1. \quad 3x^2 + 70 &= 22 \\ -70 \quad -70 \\ 3x^2 &= -48 \\ \frac{3x^2}{3} &= \frac{-48}{3} \\ \sqrt{x^2} &= \sqrt{-16} \\ X &= \pm 4i\end{aligned}$$

$$\begin{aligned}2. \quad 6 - 2x^2 &= 96 \\ -6 \quad -6\end{aligned}$$

$$\begin{aligned}-2x^2 &= 90 \\ \frac{-2x^2}{-2} &= \frac{90}{-2} \\ \sqrt{x^2} &= \sqrt{-45}\end{aligned}$$

$$\begin{aligned}\sqrt{45} \\ = \sqrt{9 \cdot 5}\end{aligned}$$

$$X = \pm 3i\sqrt{5}$$

$$3. \quad (x+7)^2 - 38 = -13$$

$$\begin{aligned}&+38 \quad +38 \\ \sqrt{(x+7)^2} &= \sqrt{25} \\ x+7 &= \pm 5 \\ \begin{aligned} \swarrow & \searrow \\ x+7 &= -5 & x+7 &= 5 \\ -7 \quad -7 & & -7 \quad -7 \\ x &= -12 & x &= -2 \end{aligned} \\ X &= -12, -2\end{aligned}$$

$$4. \quad (x - 2)^2 + 71 = 7$$

$$\begin{array}{r} -71 \quad -71 \\ \sqrt{(x-2)^2} = \sqrt{64} \end{array}$$

$$\begin{array}{r} x-2 = \pm 8i \\ +2 \quad +2 \end{array}$$

$$\boxed{x = 2 \pm 8i}$$

$$5. \quad 2(x + 1)^2 + 119 = 15$$

$$\begin{array}{r} -119 \quad -119 \\ \frac{2(x+1)^2}{2} = \frac{-104}{2} \\ \sqrt{(x+1)^2} = \sqrt{-52} \end{array} \quad \leftarrow 4 \cdot 13$$

$$x+1 = \pm 2i\sqrt{13}$$

$$\begin{array}{r} -1 \quad -1 \\ \boxed{x = -1 \pm 2i\sqrt{13}} \end{array}$$

Now you can finish Hwk #17

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simplify imaginary solutions

$$i = \sqrt{-1} = i$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

$$i^5 = i$$

$$i^6 = -1$$

$$i^7 = -i$$

$$i^8 = 1$$

$$i^9 = i$$

$$i^{10} = -1$$

$$i^{11} = -i$$

$$i^{12} = 1$$

Powers of i have a repeating pattern.
It repeats in groups of four.

Simplify each Power
of i

$$i^{18} = -1$$

$$i^{27} = -i$$

Simplify each power of i

1. $i^{37} = i$

$$\begin{array}{r} 9 \text{ R} = 1 \\ 4 \overline{) 37} \\ \underline{- 36} \\ 1 \end{array}$$

the remainder represents how far into the next pattern you are. A remainder of 1 means you are 1 into the next pattern of 4 so it's the same as i^1

2. $i^{172} = 1$

$$\begin{array}{r} 43 \\ 4 \overline{) 172} \\ \underline{- 16} \\ 12 \\ \underline{- 12} \\ 0 \end{array}$$

remainder of zero means you've just completed a pattern of 4 so it's the same as i^4

3. $i^{331} = -i$

$$\frac{331}{4} = 82.75$$

this decimal represents a remainder of 3 so it's the same as i^3

4. $i^{454} = -1$

$$\frac{454}{4} = 113.5$$

this decimal represents a remainder of 2 so it's the same as i^2