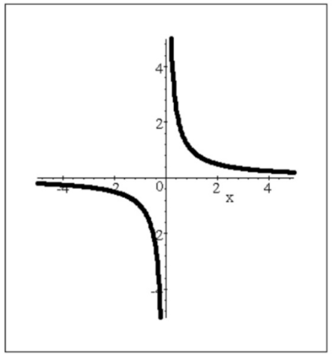
Horizontal Asymptotes:

The horizontal line that a graph approaches as you move farther and farther to the left and right. (END BEHAVIOR)

The value of y that the function approaches as x gets larger and larger (pos and neg).

The graph might come down to the HA at the end or it might rise up to the HA at the end



This is the graph of the Parent Reciprocal Function:

y = 1/x   the Horizontal Asymptote is y = 0

The left-end approaches the HA from BELOW

The right-end approaches the HA from ABOVE

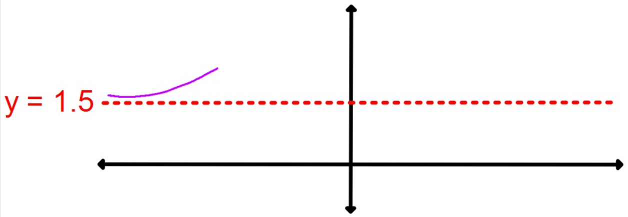
$$y = \frac{3x^2 - 4x}{13 + 2x^2}$$

The HA of this rational function is y = 1.5

Does the left-end approach the HA from above or below?

Left-end

x	y
-10	1.5962
-100	1.519
-1000	1.502
-10000	1.5002



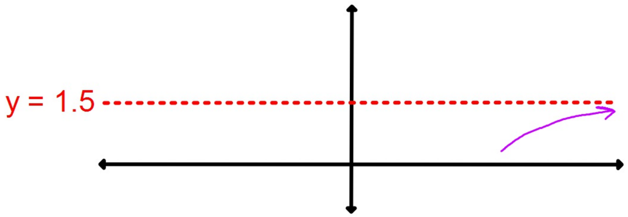
$$y = \frac{3x^2 - 4x}{13 + 2x^2}$$

The HA of this rational function is y = 1.5

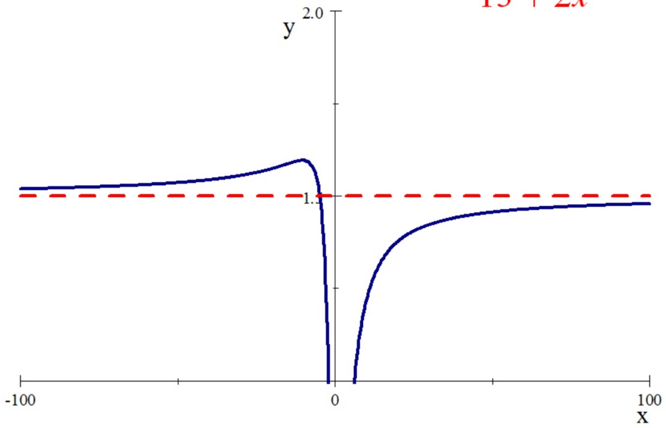
Does the right-end approach the HA from above or below?

Right-end

x	y
10	1.2207
100	1.479
1000	1.498
10000	1.4998

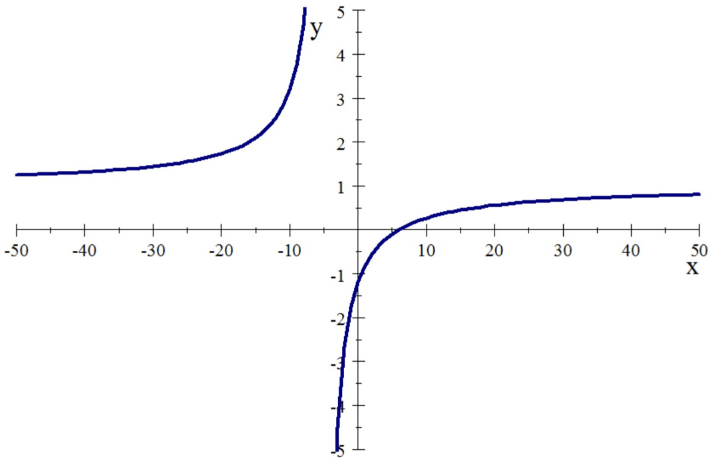


This is the actual graph of the function: $y = \dfrac{3x^2 - 4x}{13 + 2x^2}$

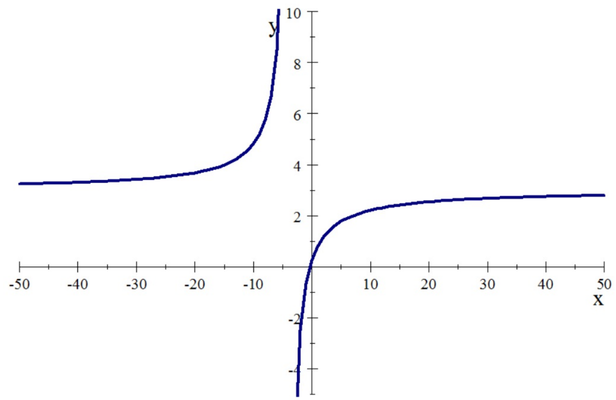


Horizontal Asymptote Exploration:

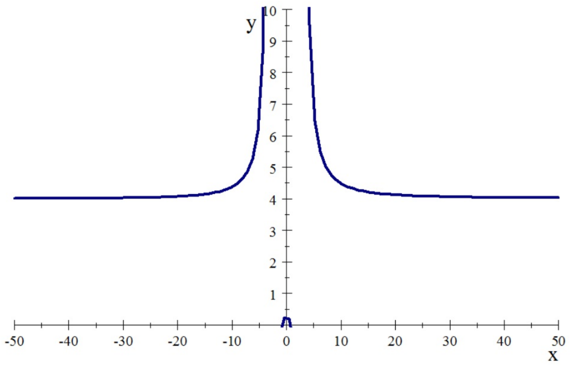
1. $y = \dfrac{x-6}{x+5}$ HA: $y=1$



2. $y = \dfrac{3x+1}{x+4}$ HA: $y=3$



3. $y = \dfrac{8x^2 + x - 6}{2x^2 - 21}$ HA: $y=4$



What do you notice in the equations that would give you the HA?

1. $y = \dfrac{x - 6}{x + 5}$ HA: y = 1

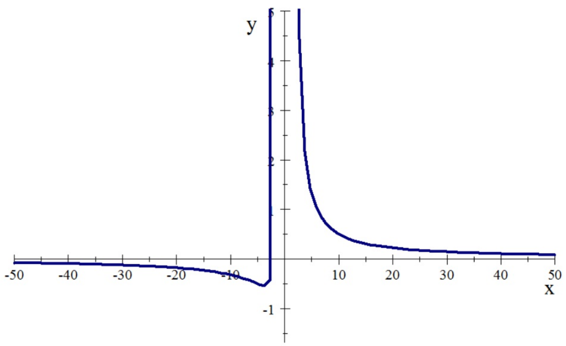
2. $y = \dfrac{3x + 1}{x + 4}$ HA: y = 3

3. $y = \dfrac{8x^2 + x - 6}{2x^2 - 21}$ HA: y = 4

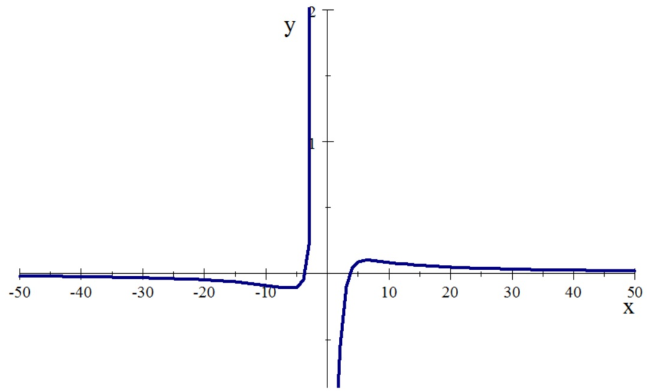
What do these three equations have in common?

The degree of the numerator and denominator are the same.

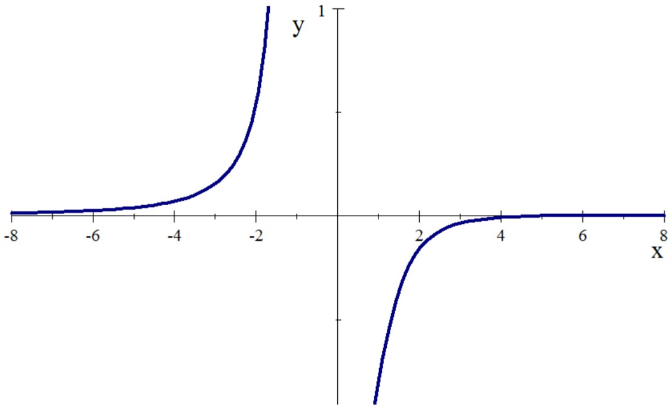
4. $y = \dfrac{4x + 9}{x^2 - 3}$ HA: $y=0$



5. $y = \dfrac{x^2 - 13}{x^3 + 7}$ HA: $y=0$



6. $y = \dfrac{x-5}{2x^3+3}$ HA: $y=0$



What do you notice in the equations that would tell you the HA is y = 0?

4. $y = \dfrac{4x+9x}{x^2-3}$ HA: y=0

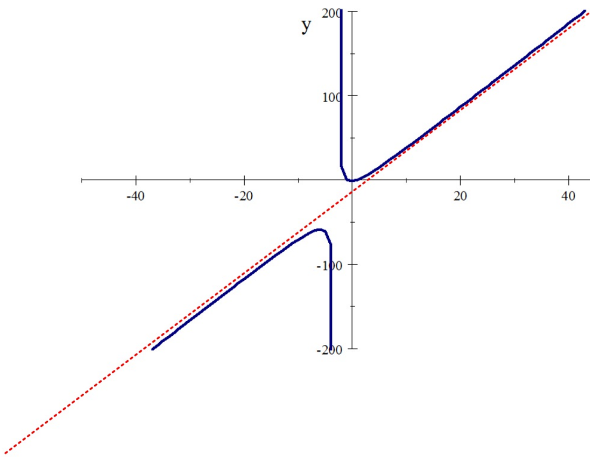
5. $y = \dfrac{x^2-13}{x^3+7}$ HA: y=0

6. $y = \dfrac{x-5}{2x^3+3}$ HA: y=0

What do these three equations have in common?

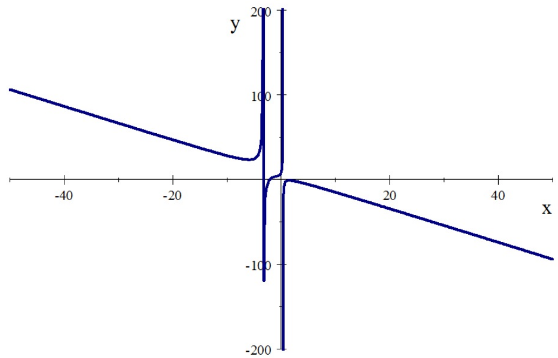
The degree of the denominator is greater than the degree of the numerator.

7. $y = \dfrac{5x^2-4}{x+3}$ HA: NONE



This graph doesn't have a Horizontal Asymptote it has what is called a Slant Aymptote

8. $y = \dfrac{-2x^3+5x-8}{x^2+3x-1}$ HA: NONE



What do you notice in the equations that would tell you that there is no HA?

7. $y = \dfrac{5x^2 - 4}{x + 3}$ HA: None

What do these three equations have in common?

The degree of the numerator is greater than the degree of the denominator.

8. $y = \dfrac{-2x^3 + 5x - 8}{x^2 + 3x - 1}$ HA: None

Without using a table or a graph how could you tell from the equation what the horizontal asymptote is or if it even has one?

Predict the Horizontal Asymptote for each of the rational functions below, if any.

a. $y = \dfrac{10x + 7}{5x - 3}$ HA: $y = 2$

b. $y = \dfrac{6x^2 - 5}{2x + 3}$ HA: NONE

c. $y = \dfrac{12x - 11}{3x^2 - 1}$ HA: $y = 0$

Horizontal Asymptotes: Depends on the degrees of the Numerator and the Denominator

Case 1: Degree of the Numerator > Degree of the Denominator

No HA

Case 2: Degree of the Numerator = Degree of the Denominator

HA: y = ratio of the Leading Coefficients

Case 1: Degree of the Denominator > Degree of the Numerator

HA: $y = 0$

7. $y = \dfrac{5x^2 - 4}{x + 3}$

This graph has what is called a Slant Asymptote.

Find this quotient

You can use Synthetic Division

$$\begin{array}{r|rrr} -3 & 5 & 0 & -4 \\ & & -15 & 45 \\ \hline & \underbrace{5 \quad -15} & & 41 \end{array}$$

these numbers represent the Quotient:

5x - 15 R=41

y=5x-15 is the equation of the slant asymptote.