Use the table function on the graphing calculator to determine the HA. Then state the left and right end-behavior of this function:

$$y = \frac{3x+7}{2x^2 - 15x - 143}$$

Left end:

as $x \to -\infty, y \to 0^-$

X	Y
-100	-0.014
-1000	-0.0015
-10000	-0.00015
-100000	-0.000015

y approaches zero from below

this shows that as the graph moves farther and farther left (bigger neg) the graph (y-value) gets closer to zero but is always below the line y=0 (still a little negative)

Right end:

as $x \to \infty, y \to 0^+$

X	Y
100	-0.017
1000	0.0015
10000	0.00015
100000	0.000015

y approaches zero from above

this shows that as the graph moves farther and farther right (bigger pos) the graph (y-value) gets closer to zero but is always above the line y=0 (still a little positive)

Using the same function, State the behavior on both sides of it's VA at: x=13

$$y = \frac{3x+7}{2x^2 - 15x - 143}$$

Left side of x=13

as $x \to 13^-, y \to -\infty$

X	Y
12.9	-12.42
12.99	-124.3
12.999	-1243
12.9999	-12432

As x gets closer to 13, from the left, y gets bigger and bigger negative(it goes down)

Right side of x=13

as $x \to 13^+, y \to \infty$

X	Y
13.1	12.45
13.01	124.34
13.001	1243.3
13.0001	12432

As x gets closer to 13, from the right, y gets bigger and bigger positive(it goes up)

When the denominator of a rational function is zero the function is undefined.

Because this value of x can never be used this leads to a break in the graph (it's not continuous)

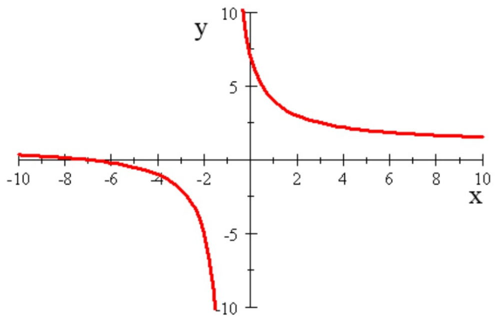
These breaks in the graph are one of two types:

Vertical Asymptotes

Holes

Graph the rational function f(x) in a standard window.

$$f(x) = \frac{x+7}{x+1}$$



There is a break in the graph at x = -1

This kind of break in the graph is called a Vertical Asymptote

as a graph approaches a vertical asymptote it will either increase without bound (go up) or decrease without bound (go down)

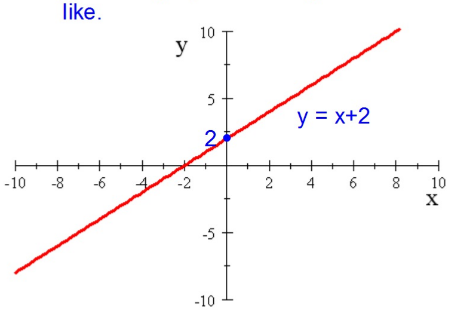
Graph the rational function f(x) in a standard window.

$$f(x) = \frac{(\cancel{(x-3)}(x+2))}{\cancel{(x-3)}}$$

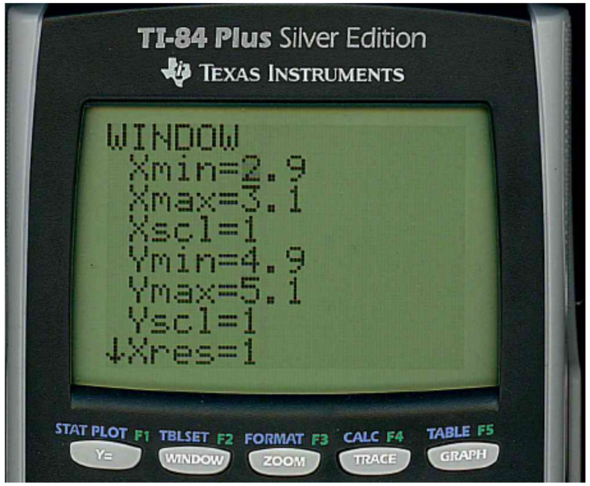
Why do you think that there isn't a vertical asymptote at x = 3?

Because the factor that created the point of discontinuity at x=3 cancels.

When simplified the orginal function becomes f(x) = x+2 which is what the graph of the original looks like.



Change the window to the following:

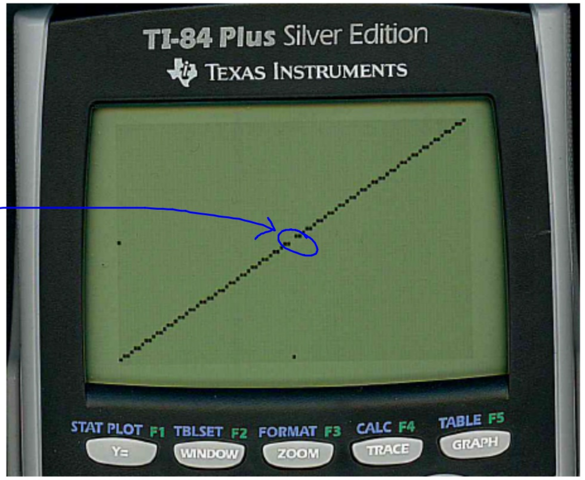


What do you see?

This kind of break in the graph is called a **Hole**

$$f(x) = \frac{((x-3)(x+2))}{(x-3)}$$

There is a HOLE when x=3



Why did this graph have a Vertical Asymptote at x = -1

$$f(x) = \frac{x+7}{x+1}$$

x=-1 is a zero of ONLY the denominator

but

this graph had a hole at x = 3?

$$f(x) = \frac{(x-3)(x+2)}{(x-3)}$$

x=3 is a zero of both the numerator and denominator

Breaks in the graph are caused by zeros of the denominator and are called:

Points of Discontinuity

Holes or **Vertical Asymptotes**

Occur at values of x that are zeros of $\boxed{\text{both}}$ the denominator AND numerator

Occur at values of x that are zeros of $\boxed{\text{the denominator ONLY.}}$

Another way to say this is that the Numerator and Denominator have a factor in common.

An exception to this rule:

$$y = \frac{x^2 - 16}{x^2 + 8x + 16} = \frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(x+4)}$$

X= -4 is a VA

This result is more like

$$f(x) = \frac{x+7}{x+1}$$

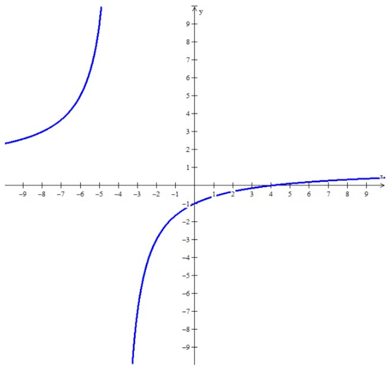
Which had a VA

It's less like

$$f(x) = \frac{(x-3)(x+2)}{(x-3)}$$

Because in the case above the (x-3) cancels and it doesn't appear in the denominator anymore.

$$y = \frac{x^2 - 16}{x^2 + 8x + 16} = \frac{(x+4)(x-4)}{(x+4)(x+4)}$$



Why is there a VA at x = -4 and not a hole?

Even though the factors (x+4) are common to the numerator and denominator, when you cancel them there is still (x+4) left in the denominator.

Properties **Vertical Asymptotes**

The rational function $f(x) = \frac{P(x)}{Q(x)}$ has a point of discontinuity for each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have no common real zeros, then the graph of $f(x)$ has a vertical asymptote at each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have a common real zero a, then there is a hole in the graph or a vertical asymptote at $x = a$.

Find any points of dicontinuity and classify them as Vertical Asymptotes or Holes.

1. $y = \frac{2x(x+1)\cancel{(x-6)}}{(x-1)\cancel{(x-6)}}$

Pts of Discontinuity: 1, 6

VA: $x = 1$

Holes: $x = 6$

2. $y = \frac{3x^2 - 6}{x^2 - 4} = \frac{3(x^2-2)}{(x+2)(x-2)}$

Pts of Discontinuity: $x = -2, 2$

VA: $x = -2, 2$

Holes: NONE

3. $y = \frac{x^2 - x - 12}{x^2 - 16}$

$$\frac{\cancel{(x-4)}(x+3)}{(x+4)\cancel{(x-4)}}$$

Pts of Discontinuity: $x = \pm 4$

VA: $x = -4$

Holes: $x = 4$

4. $y = \frac{x^2 + 6x + 9}{x^2 + 5x + 6}$

$$\frac{(x+3)\cancel{(x+3)}}{(x+2)\cancel{(x+3)}}$$

Pts of Discontinuity: $x = -2, -3$

VA: $x = -2$

Holes: $x = -3$

5. $y = \frac{2x^2}{x^2 + 3}$

Pts of Discontinuity:

There are no points of discontinuity because the denominator will never be equal to zero (it has no real zeros!)

VA:

Holes:

You can now finish:

Hwk #5 Sec 9-3

Pages 505

Problems 2, 3, 5, 12, 13, 17, 18,