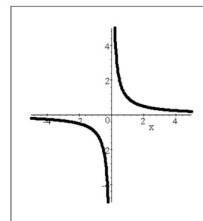The graph of $y = \dfrac{1}{X}$



**Vertical Asymptote**

the y-axis
EQ: x=0

**Horizontal Asymptote**

the x-axis
EQ: y=0

---

The graph of $y = \dfrac{1}{X}$



Describe the location of the two branches of the Parent Function.

Quadrants I and III (relative to the asymptotes)

---

$y = \dfrac{k}{X}$

**k is pos:**

Branches are in the
1st and 3rd Quadrants

**k is neg:**

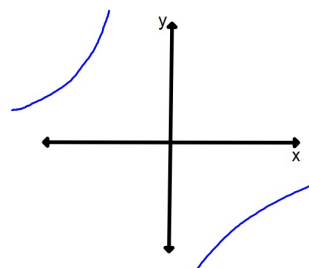Branches are in the
2nd and 4th Quadrants

**k is large:**

Branches are further
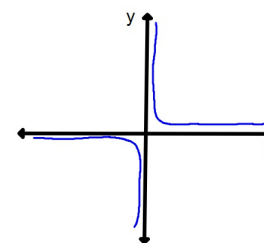from the origin

**k is small:**

Branches are closer to
the origin

---

Without using a graphing calculator sketch the graph of each:
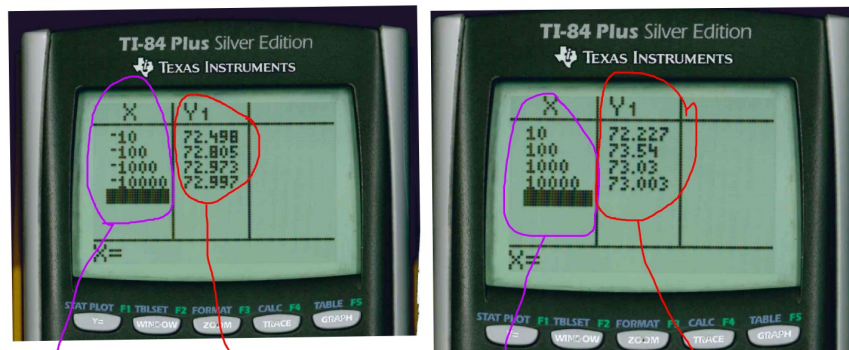
$$y = \dfrac{-20}{x}$$



$$y = \dfrac{0.3}{x}$$

**What do these two calculator screens tell you?**

The farther from the origin you are (both left and right) the closer the function gets to 73......the graph flattens out and approaches the horizontal line y = 73



as you move farther to the left,

y gets closer to 73

as you move farther to the right,
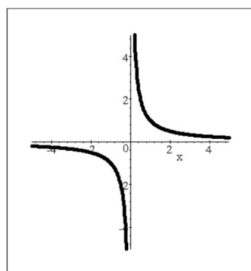
y gets closer to 73

---

At the ends of a graph with a Horizontal Asymptote(HA) the graph will get very close to that y-value (HA) but either be

barely greater than that value........approaches the HA from above

or

barely less than that value........approaches the HA from below.

---

Notation for the end-behavior of this graph.



Left-end:

as $x \longrightarrow -\infty$ , $y \longrightarrow 0^{-}$

Right-end:

as $x \longrightarrow \infty$, $y \longrightarrow 0^{+}$

---

**What does this table show about the behavior of the graph of the Parent Reciprocal Function near the Vertical Asymptote x=0?**

| X | Y |
|---|---|
| -0.1 | -10 |
| -0.001 | -1000 |
| -0.001 | -1000 |
| -0.0001 | -10000 |
| 0.1 | 10 |
| 0.01 | 100 |
| 0.001 | 1000 |
| 0.0001 | 10000 |

As the graph approaches the VA from the left.......
the Function decreases without bound

As x gets smaller and smaller negative (to the left of zero)
y gets bigger and bigger negative (goes down)

As the graph approaches the VA from the right.......
the Function increases without bound

As x gets smaller and smaller positive (to the right of zero)
y gets bigger and bigger positive (goes up)

When a graph approaches a Vertical Asymptote (VA) the graph will either increase without bound the closer to the VA you get or it will decrease without bound the closer to the VA you get
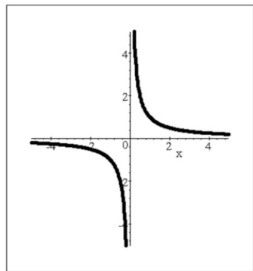


Notation for behavior around the VA x=0 this graph.

To the left of x=0:

as $x \longrightarrow 0^-$, $y \longrightarrow -\infty$

To the right of x=0:

as $x \longrightarrow 0^+$, $y \longrightarrow \infty$

$$Y_1 = \frac{28.6}{x - 47} - 81$$

What do you think the Vertical and Horizontal Asymptotes of this function are?

HA  $y = -81$

| left end: | | right end: | |
|---|---|---|---|
| x | y | x | y |
| -100 | -81.19 | 100 | -80.46 |
| -1000 | -81.03 | 1000 | -81.97 |
| -10000 | -81.0028 | 10000 | -80.997 |

Both ends of the graph are approaching y= -81 so this is the HA.

VA  $x = 47$ ← the zero of the denominator

$$y = a(x - h)^2 + k \qquad y = a|x - h| + k$$

a: Vertical Stretch or Shrink Factor
   if a<0 there is an x-axis reflection (Upside Down)

h: Horizontal Translation

these lead to  vertex $(h, k)$

k: Vertical Translation

$$y = \frac{a}{x - h} + k$$

The larger a is... the farther the branches are from the origin
The smaller a is...the closer the branches are to the origin

a: Vertical Stretch or Shrink Factor
   if a<0 there is an x-axis reflection (Upside Down)

a>0: branches are in Quadrants I & III
a<0: branches are in Quadrants II & IV

h: Horizontal Translation

   Vertical Asymptote becomes:   x = h

k: Vertical Translation

   Horizontal Asymptote becomes:   y = k

What are the equations of the two asymptotes for each reciprocal function?

**1.** $y = \dfrac{30}{x-7} + 2$   2 up

7 RIGHT

HA: $y = 2$

VA: $x = 7$

**2.** $y = \dfrac{-0.3}{x+5} - 8$   8 down

5 Left

HA: $y = -8$

VA: $x = -5$

---

Write an equation for the translation of $y = \frac{3}{x}$ that has the given asymptotes.

1. $y = 4$ and $x = -3$
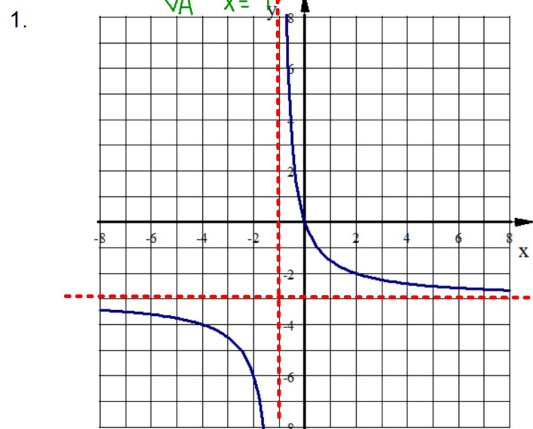
4 up    3 Left

$y = \dfrac{3}{x+3} + 4$

2. $y = 0$ and $x = 9$

NO Vert trans       9 RT

$y = \dfrac{3}{x-9}$

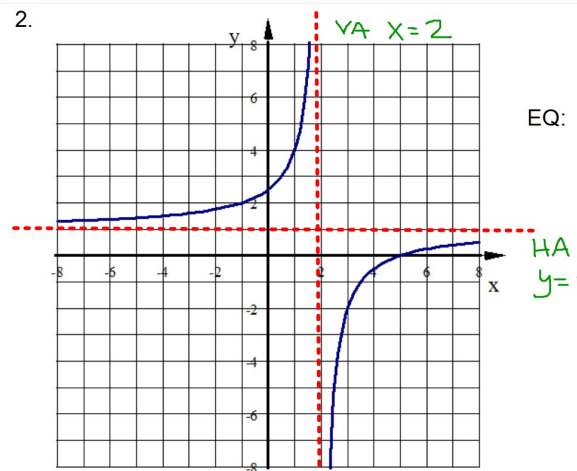3. $y = -5$ and $x = 0$

5 down    NO Horiz trans

$y = \dfrac{3}{x} - 5$

---

For 1 and 2, write the equation of each graph which are tranformations of the equation: $y = \frac{3}{x}$

**1.**

VA   $x = -1$



HA
$y = -3$

1 Left   3 down

EQ: $y = \dfrac{3}{x+1} - 3$

---

**2.**

VA $x = 2$
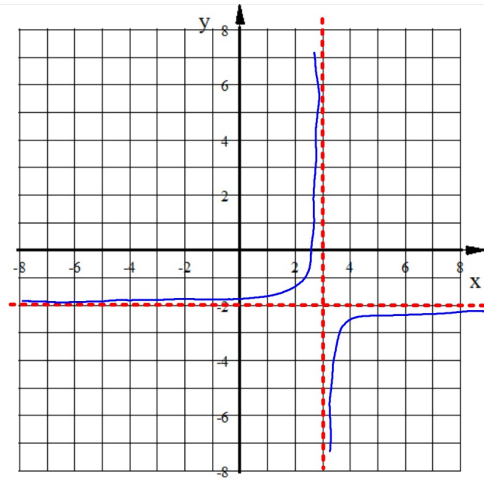


HA
$y = 1$

2 RIGHT  1 up

EQ: $y = \dfrac{-3}{x-2} + 1$

For 3 and 4, sketch the graph of each.
Show asymptotes as dashed lines.

3. $y = \dfrac{-0.1}{x-3} - 2$

3 RT

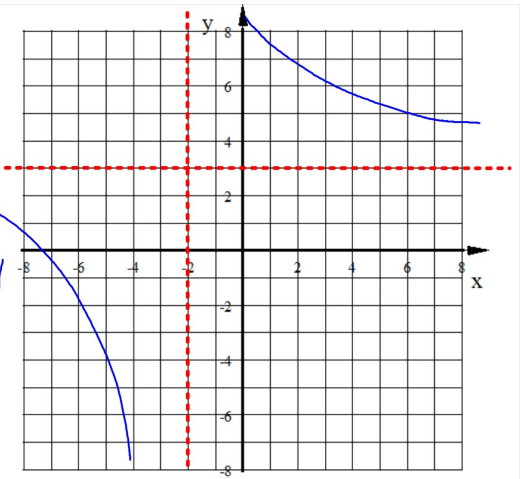Branches
in
Quad
II & IV
close
to
asymptotes

VA: $X=3$

2 down

HA: $y=-2$



4. $y = \dfrac{15}{x+2} + 3$

2 Left

3 up

VA: $X=-2$

HA: $y=3$

Branches
are in Quad I & III
branches are
far from
where asymptotes
cross



You can now finish Hwk #4

Practice Sheet      Sec 9-2

Graphs of Reciprocal Functions