Find this quotient.

$$\frac{x^3+3x^2-x+4}{x-2}$$

Using Synthetic Division:

2	1	3	-1	4
		2	10	18
	1	5	9	22

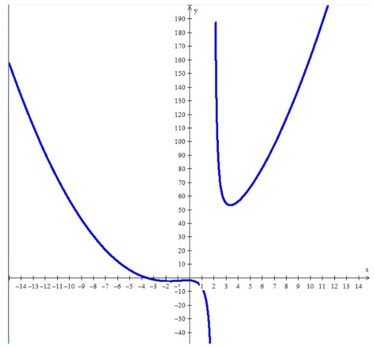
This represents the quotient:

$x^2 + 5x + 9$ R=22

Graph this rational function in the following window:

x [-15,15]
y [-50,200]

$$y = \frac{x^3+3x^2-x+4}{x-2}$$



What kind of asymptote does this graph have?

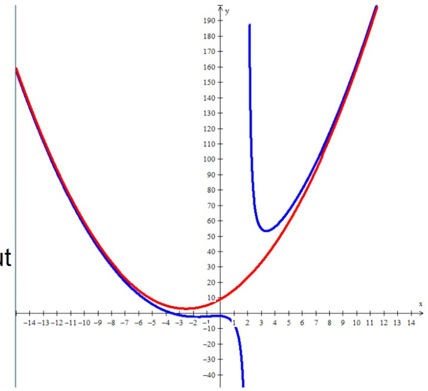
Quadratic

Graph the original function in Y_1

$$y = \frac{x^3+3x^2-x+4}{x-2}$$

Graph the quotient, without the remainder, in Y_2

$x^2 + 5x + 9$



The end behavior of this graph is a quadratic. The equation of this end-behavior is the quotient of the original rational function, without the remainder.

x and y intercepts of Rational Functions:

Y-Intercepts: the result of replacing x with zero.

Find the y-intercepts of each function.

1. $y = \frac{x^2-9x+20}{x^2+7x+10}$

$$y = \frac{(0)^2 - 9(0) + 20}{(0)^2 + 7(0) + 10} = \frac{20}{10}$$

y-int: $= 2$

2. $y = \frac{3x^2-7x}{x^2+x-2}$

the denominator becomes zero when you substitute zero for x which means the function is undefined for that value of x.

y-int: None

3. $y = \dfrac{x^2 - 4}{2x^2 + 6x} = \dfrac{(0)^2 - 4}{2(0)^2 + 6(0)} = \dfrac{-4}{0}$

y-int: NONE

When you replace x with zero the denominator equals zero and the ratio becomes undefined. Therefore, there is no y-int. When you factor the rational function you also notice that there is a Vertical Asymptote when x=0. This means that when the y-axis is a Vertical Asymptote there is no y-intercept.

$$\frac{(x+2)(x-2)}{2x(x+3)}$$

VA are x=0 and x=-3

In general, if a rational function has a y-intercept, it is the:

Ratio of the Constants

A graph can have at most ONE y-intercept.

X-Intercepts: the result of replacing y with zero.

This means you are setting the ratio equal to zero and solving for x.

The only way a fraction equals zero is if the NUMERATOR equals zero.

In general, the x-intercepts of a Rational Function are the:

Zeros of the numerator (as long as they don't match zeros of the denominator)

A graph can have multiple x-intercepts.

find the x-intercepts of each function.

1. $y = \dfrac{x^2 - 9x + 20}{x^2 + 7x + 10}$

x-int: 4, 5

$$\frac{(x-5)(x-4)}{(x+5)(x+2)}$$

2. $y = \dfrac{x^2 - 4}{3x^2 - 6x}$

x-int: $= \cancel{2}, -2$

$$\frac{(x+2)(x-2)}{3x(x-2)}$$

3. $y = \dfrac{4x^2 + 8}{3x^2 - 2x + 1}$

x-int: NONE

$4x^2$ + 8 will never equal zero
therefore, there will be NO x-int.

You can now finish Hwk #6.

Practice Sheet: Horizontal Asymptotes and x & y-intercepts

Due Monday

Let's put this all together.
Graph the following Rational Function showing:

- All asymptotes as dashed lines
- X & Y-intercepts, if any
- Correct behavior around each asymptote.

$$y = \frac{x^2 - x - 12}{x^2 + 6x - 7} = \frac{(x-4)(x+3)}{(x+7)(x-1)}$$

$$y = \frac{x^2 - x - 12}{x^2 + 6x - 7} = \frac{(x-4)(x+3)}{(x+7)(x-1)}$$

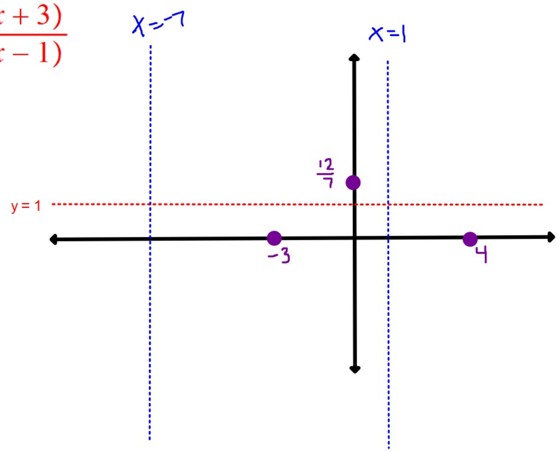
x-int: $-3, 4$

y-int: $\frac{12}{7}$

VA: $x = -7, 1$

HA: $y = 1$

Put all of this on a graph.



$$y = \frac{x^2 - x - 12}{x^2 + 6x - 7} = \frac{(x-4)(x+3)}{(x+7)(x-1)}$$

connect the intercepts without crossing the Vertical Asymptotes.

Typical behavior is when a graph goes down on one side of a VA it tends to go up on the other side.

Remember, you can't cross the x or y axes anywhere but the intercepts.

Remeber to flatten out at the ends of the graph and get close to the HA.