

Graph to find all REAL solutions.

$$x^3 - 4x^2 + x + 26 = 0 \rightarrow x = -2$$

since -2 is a real solution $(x+2)$ is a factor of the original polynomial.

If $x+2$ is a factor use polynomial division to find another factor.

$$\begin{array}{r} x^2 - 6x + 13 \\ x+2 \overline{) x^3 - 4x^2 + x + 26} \\ \underline{-(x^3 + 2x^2)} \\ -6x^2 + x \\ \underline{-(6x^2 + 12x)} \\ 13x + 26 \\ \underline{-(13x + 26)} \\ 0 \end{array}$$

Find the zeros of this other factor to get the remaining 2 solutions.

(use Quadratic Formula)

or completing the sq.

$$x^2 - 6x + 13$$

$$b^2 - 4ac = -16$$

$$x = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

Therefore, the three solutions to the original cubic are -2(real) and $3 \pm 2i$ (imaginary)

-1 and 2 are solutions of this equation. The other two solutions are imaginary.

Find the other two solutions.

$$x^4 - x^3 + 2x^2 - 4x - 8 = 0$$

$$\begin{array}{r} x^3 - 2x^2 + 4x - 8 \\ x+1 \overline{) x^4 - x^3 + 2x^2 - 4x - 8} \\ \underline{-(x^4 + x^3)} \\ -2x^3 + 2x^2 \\ \underline{-(2x^3 + 2x^2)} \\ 4x^2 - 4x \\ \underline{-(4x^2 + 4x)} \\ -8x - 8 \\ \underline{-(-8x - 8)} \\ 0 \end{array}$$

turn these two zeros into the following factors: $(x+1)$ & $(x-2)$. You can now divide the original equation by one of these factors:

At this point we could take this quotient, $x^3 - 2x^2 + 4x - 8$ and divide by the other factor, $x-2$, or we could try to factor this cubic:

$$\begin{array}{cc} x & -2 \\ x^2 & \boxed{\begin{array}{cc} x^3 & -2x^2 \\ +4x & -8 \end{array}} \end{array}$$

If we did divide $x^3 - 2x^2 + 4x - 8$ by $x-2$ we would end up with $x^2 + 4$ as well.

We now have the following factors of the the original problem:

$$(x+1)(x-2)(x^2+4)$$

These factors lead to the following four solutions: $x = -1, 2, \pm 2i$