

Given the following function:

$$f(x) = 5x^3 - 28x^2 + 25x + 41$$

Find  $f(4) = 5(4)^3 - 28(4)^2 + 25(4) + 41 = 13$

#### Theorem

#### Remainder Theorem

If a polynomial  $P(x)$  of degree  $n \geq 1$  is divided by  $(x - a)$ , where  $a$  is a constant, then the remainder is  $P(a)$ .

this means that the dividend must be at least a linear function.

The remainder of a quotient can be found by evaluating the dividend using the zero of the divisor.

$$\frac{5x^3 - 28x^2 + 25x + 41}{x - 4}$$

$5x^3$	$-28x^2$	$+25x$	$+41$
$-20x^3$	$+12x^2$	$-100x$	$+164$
	$+32x^2$	$-75x$	$-123$
		$+32x$	$-82$
			$+13$

What is the remainder? 13

What is the zero of the divisor? 4

Evaluate the dividend using the zero of the divisor: 13

Do you notice anything? the remainder is the same number that you get when you evaluate the dividend with the zero of the divisor.

Find the remainder to this quotient:  $\frac{2x^3 + x^2 - 7x - 10}{x - 2}$

<p>Remainder Thm</p> $2(2)^3 + (2)^2 - 7(2) - 10 = -4$	<p>Divide</p> $\begin{array}{r} 2x^2 + 5x + 3 \\ x-2 \overline{) 2x^3 + x^2 - 7x - 10} \\ \underline{2x^3 - 4x^2} \phantom{- 10} \\ 5x^2 - 7x - 10 \\ \underline{5x^2 - 10x} \phantom{- 10} \\ 3x - 10 \\ \underline{- 3x + 6} \\ -4 \end{array}$
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Using either method, the remainder is -4

What is the remainder of this quotient?

$$\frac{6x^2 + 5x - 2}{x - 4}$$

You could answer this questions by:

1. Actually doing the division

OR

2. Using the Remainder Theorem

zero of the  
divisor = 4

$$6(4)^2 + 5(4) - 2 = 96 + 20 - 2 = 114$$

The remainder = 114

Is  $x - 3$  a factor of  $2x^3 - 12x^2 + 21x - 9$  ?

Only if the remainder is zero!

$$2(3)^3 - 12(3)^2 + 21(3) - 9$$
$$54 - 108 + 63 - 9 = 0$$

Yes,  $x-3$  is a factor because the remainder is zero.

Is  $x + 2$  a factor of  $x^3 + 7x^2 + 3x - 21$  ?

$$(-2)^3 + 7(-2)^2 + 3(-2) - 21$$

$$-8 + 28 - 6 - 21 \neq 0$$

No,  $x+2$  is not a factor because the remainder isn't zero.

Now you can finish Hwk #31      Sec 6-3

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Problems 11, 12, 23, 28, 29, 43, 44

Use polynomial  
long division or  
divide using the "Box"

Use just the  
Remainder Theorem

## Synthetic Division

Uses the zero of the divisor.  
By reversing the sign of the divisor you can ADD throughout the process instead of subtracting.

Synthetic Division can only be used if the divisor is linear and the leading coefficient is 1.

Meaning either  $\div(x + a)$  or  $\div(x - a)$

$$\frac{x^3 - 2x^2 - 31x + 20}{x + 5} = x^2 - 7x + 4 \quad R=0$$

Zero of the Divisor

Coefficients of the dividend in Standard Form

-5	1	-2	-31	20	
		-5	35	-20	
	1	-7	4	0	

Bring down the first #

Multiply and ADD

$(-5)(1)$

$-2+5$

$(-5)(-7)$

$-31+35$

$(-5)(4)$

$20+20$

Find each quotient using Synthetic Division

1.  $\frac{4x^3 - 6x^2 - 7x - 33}{x - 3} = 4x^2 + 6x + 11 \quad R=0$

3	4	-6	-7	-33
		12	18	33
	4	6	11	0

2.  $\frac{2x^4 + 18x^3 + 34x^2 + 43x + 10}{x + 7} = 2x^3 + 4x^2 + 6x + 1 \quad R=3$

-7	2	18	34	43	10
		-14	-28	-42	-7
	2	4	6	1	3