$$x^4+3x^3+x^2+15x-20=0$$

Find all four Complex Solutions (real and imaginary) by doing the following:

a. Find all real solutions by graphing then, Real solutions are 1, -4

Therefore, factors of the original polynomial are (x-1) and (x+4)

b. Use the real zeros to find the remaining imaginary solutions using division.

$$
\begin{array}{r|l}
 & x^3+4x^2+5x+20 \\
x-1 & x^4+3x^3+x^2+15x-20 \\
 & -(x^4-x^3) \\
 & 4x^3+x^2 \\
 & -(4x^3-4x^2) \\
 & 5x^2+15x \\
 & 5x^2-5x \\
 & 20x-20 \\
 & 20x-20 \\
 & 0
\end{array}
$$

this quotient is, therefore, another factor of the original dividend.

after dividing by x-1 you could take that resulting quotient and divide by the other factor (x+4):

$$
\begin{array}{r|l}
 & x^2+5 \\
x+4 & x^3+4x^2+5x+20 \\
 & -(x^3+4x^2) \\
 & 5x+20 \\
 & 5x+20 \\
 & 0
\end{array}
$$

$(x-1)(x+4)(x^2+5)$

$x=1,-4,\ \pm i\sqrt{5}$

OR

after dividing by x-1 you could try factoring the quotient $x^3+4x^2+5x+20$ by using the box:

	x	$+4$
x^2	x^3	$+4x^2$
$+5$	$+5x$	$+20$

This leads to the same factors

$(x-1)(x+4)(x^2+5)$

which will lead to the same solutions:

$x=1,-4,\ \pm i\sqrt{5}$

another option: if these are factors: (x-1) and (x+4)

Then (x-1)(x+4) = x^2+3x-4 is also a factor.

This means you could divide the original dividend by the product of the two original factors:

$$
\begin{array}{r|l}
 & x^2+5 \\
x^2+3x-4 & x^4+3x^3+x^2+15x-20 \\
 & -(x^4+3x^3-4x^2) \\
 & 5x^2+15x-20 \\
 & -(5x^2+15x-20) \\
 & 0
\end{array}
$$

this will lead to the same third factor x^2+5

$(x-1)(x+4)(x^2+5)$

which will lead to the same solutions:

$\boxed{x=1,-4,\ \pm i\sqrt{5}}$

Using the "Box" for Polynomial Division

Expand using the Box:

$(x+3)(x^2-2x+5)$

	x^2	$-2x$	$+5$
x	x^3	$-2x^2$	$5x$
$+3$	$3x^2$	$-6x$	$+15$

x^3+x^2-x+15

$$
\begin{array}{r|l}
 & 2x^2-6x+3 \\
x+4 & 2x^3+2x^2-21x+12
\end{array}
$$

$(x+4)(\qquad)=2x^3+2x^2-21x+12$

	$2x^2$	$-6x$	$+3$	
x	$2x^3$	$-6x^2$	$+3x$	0
$+4$	$8x^2$	$-24x$	12	

$2x^3+2x^2-21x+12=\boxed{(x+4)(2x^2-6x+3)}$

Use the "Box" for Polynomial Division

$3x - 2 \overline{)\, 12x^3 - 5x^2 - 17x + 8}$ $= 4x^2 + x - 5 \quad R = -2$

or

$4x^2 + x - 5 - \frac{2}{3x-2}$

	$4x^2$	$+x$	-5	
$3x$	$12x^3$	$+3x^2$	$-15x$	-2
-2	$-8x^2$	$-2x$	10	

Use the "Box" for Polynomial Division

$$\frac{x^2 + 8x - 13}{x - 3}$$

	x	$+11$	
x	x^2	$11x$	$+20$
-3	$-3x$	-33	

remainder

$= x + 11 \quad R = 20$

or

$x + 11 + \frac{20}{x-3}$

If $x + 2$ is a factor, use polynomial division to factor $x^3 + x^2 - 22x - 40$ completely.

Since you are trying to factor a polynomial with four terms you could try using the box right away.

Remember, as a check when you are factoring using the box is that the constants of your factors (+1 and -22) should multiply to the constant inside the box (-40). Since (+1)x(-22) doesn't equal -40, this polynomial can't be factored using the box.

	x	$+1$
x^2	x^3	x^2
-22	$-22x$	-40

This attempt at using the box doesn't work!

If $x + 2$ is a factor, use polynomial division to factor $x^3 + x^2 - 22x - 40$ completely.

Since factoring using the box doesn't work try doing polynomial division.

	x^2	$-x$	-20	
x	x^3	$-x^2$	$-20x$	0
$+2$	$2x^2$	$-2x$	-40	

$x^3 + x^2 - 22x - 40 = (x+2)(x^2 - x - 20)$

-20

$-5 \quad 4$

-1

$x^3 + x^2 - 22x - 40 = (x+2)(x-5)(x+4)$

Given $f(x) = x^3 - 5x^2 + 8x - 11$

Find $f(-3) = (-3)^3 - 5(-3)^2 + 8(-3) - 11 = -107$