Find this product by hand: A • B

$$A\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \quad B\begin{bmatrix} 5 & -1 \\ 2 & 10 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 7 & -43 \\ 9 & 19 \end{bmatrix}$$

a $(3\cdot5)+(-4\cdot2)=7$

b $(3\cdot-1)+(-4\cdot10)=-43$

c $(1\cdot5)+(2\cdot2)=9$

d $(1\cdot-1)+(2\cdot10)=19$

Find this product A • B

$$A\begin{bmatrix} 7 & 2 \\ -2 & 1 \\ 0 & 3 \end{bmatrix} \quad B\begin{bmatrix} 5 & -1 & 3 \\ 10 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 55 & -3 & 23 \\ 0 & 4 & -5 \\ 30 & 6 & 3 \end{bmatrix}$$

3×2 $\quad 2\times3$ $\quad 3\times3$

Find this product : B A

$$A\begin{bmatrix} 7 & 2 \\ -2 & 1 \\ 0 & 3 \end{bmatrix} \quad B\begin{bmatrix} 5 & -1 & 3 \\ 10 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 37 & 18 \\ 66 & 25 \end{bmatrix}$$

3×2 $\quad 2\times3$ $\quad 2\times2$

Matrix Division:

Dividing

And what about division? Well we **don't** actually divide matrices, we do it this way:

$A/B = A \times (1/B) = A \times B^{-1}$

where $\mathbf{B}^{-1}$ means the "inverse" of B.

So we don't divide, instead we **multiply by an inverse**.

Definition **Multiplicative Identity Matrix**

For an $n \times n$ square matrix, the **multiplicative identity matrix** is an $n \times n$ square matrix I, or I_n, with 1's along the main diagonal and 0's elsewhere.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{and so forth}$$

Use A $\begin{bmatrix} 6 & -9 \\ 4 & 5 \end{bmatrix}$ to show that I_2 $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity matrix

$$\begin{bmatrix} 6 & -9 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$a = 6 \cdot 1 + -9 \cdot 0 = 6$

$b = 6 \cdot 0 + -9 \cdot 1 = -9$

$c = 4 \cdot 1 + 5 \cdot 0 = 4$

$d = 4 \cdot 0 + 5 \cdot 1 = 5$

$$\begin{bmatrix} 6 & -9 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -9 \\ 4 & 5 \end{bmatrix}$$

this shows that $A \bullet I_2 = A$

Find each.

a. $5 \cdot 5^{-1} = 1$ b. $-12 \cdot (-12)^{-1} = 1$

5^{-1} and $(-12)^{-1}$ are called

Multiplicative Inverses of 5 and -12, respectively.

Solve this equation without using division or fractions.

$6^{-1} \cdot 6x = 45 \cdot 6^{-1}$

$x = 7.5$

You can multiply both sides by the inverse of 6 to get the same answer you would get if you had divided both sides by 6 or as if you had multiplied both sides by 1/6.

Definition **Multiplicative Inverse of a Matrix**

If A and X are $n \times n$ matrices, and $AX = XA = I$, then X is the multiplicative inverse of A, written A^{-1}.

$$AA^{-1} = A^{-1}A = I$$

Identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

AX = XA = *I* means that when you multiply the two matrices A and X you get the Identity Matrix as a result.

Show that matrices A and B are inverses.

A $\begin{bmatrix} 4 & -2 \\ -9 & 5 \end{bmatrix}$ B $\begin{bmatrix} 2.5 & 1 \\ 4.5 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Since both AB and BA lead to the Identity Matrix they must be inverses.

Are these two matrices inverses?

A $\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$ B $\begin{bmatrix} \frac{1}{3} & -4 \\ 1 & \frac{1}{2} \end{bmatrix}$

AB = $\begin{bmatrix} 5 & -10 \\ 1.67 & 5 \end{bmatrix}$

Since AB doesn't equal the Identity Matrix they are NOT inverses.

Are these two matrices inverses?

A $\begin{bmatrix} 6 & 10 \\ 5 & 8 \end{bmatrix}$ B $\begin{bmatrix} -4 & 5 \\ 2.5 & -3 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Yes, since both AB and BA equal the identity matrix these matrices MUST be inverses.

A $\begin{bmatrix} 6 & 10 \\ 5 & 8 \end{bmatrix}$

Find A^{-1}

To find the inverse of a matrix you can use a graphing calculator to do the following:

$\boxed{\text{2nd}}$ $\boxed{x^{-1}}$ (matrix) Choose this option: Name 1:[A]

You will see the following on the home screen: [A]

then press $\boxed{x^{-1}}$, you will see the following on the home screen: $[A]^{-1}$

Now press enter and you will get the inverse of A: $\begin{bmatrix} -4 & 5 \\ 2.5 & -3 \end{bmatrix}$