A $\begin{bmatrix} 8 & 5 \\ -3 & 7 \end{bmatrix}$ B $\begin{bmatrix} 10 & 3 & 9 \\ 17 & -5 & 1 \end{bmatrix}$ C $\begin{bmatrix} 4 & 20 \\ 0 & 2 \\ -1 & 6 \end{bmatrix}$ D $\begin{bmatrix} 7 & 13 & 8 \\ -1 & 4 & 10 \end{bmatrix}$

1. Which matrices can be added? B & D

2. Which can be subtracted? B & D

In order to add and subtract matrices they must have the same dimensions.

3. Is matrix addition and subtraction commutative?

Yes

No, unless you change subtraction into adding the opposite.

Enter these matrices on the graphing calculator.

A $\begin{bmatrix} 8 & 5 \\ -3 & 7 \end{bmatrix}$ B $\begin{bmatrix} 10 & 3 & 9 \\ 17 & -5 & 1 \end{bmatrix}$ C $\begin{bmatrix} 4 & 20 \\ 0 & 2 \\ -1 & 6 \end{bmatrix}$ D $\begin{bmatrix} 11 & -4 & -8 \end{bmatrix}$

4. Experimenting with the graphing calculator find which matrices can be multiplied.

5. Write down the ones that can be multiplied, with their dimensions, and the dimensions of the resulting matrix.

see next page for answers.

A $\begin{bmatrix} 8 & 5 \\ -3 & 7 \end{bmatrix}$ B $\begin{bmatrix} 10 & 3 & 9 \\ 17 & -5 & 1 \end{bmatrix}$ C $\begin{bmatrix} 4 & 20 \\ 0 & 2 \\ -1 & 6 \end{bmatrix}$ D $\begin{bmatrix} 11 & -4 & -8 \end{bmatrix}$

A • B = 2x3
2x2 2x3

B • C = 2x2
2x3 3x2

C • A = 3x2
3x2 2x2

C • B = 3x3
3x2 2x3

D • C = 1x2
1x3 3x2

To Add and Subtract two matrices they must have the exact same dimensions.

To multiply two matrices

The number of columns in the first matrix must match the number of rows in the second matrix.

The middle numbers must match

(3 x 2)(2 x 3) or (2 x 3)(3 x 2)

the dimensions of the answer are the first and last numbers of the two matrices being multiplied. Or, "the outside two #'s"

(3 x 2)(2 x 3) = 3x3 or (2 x 3)(3 x 2) = 2x2

State all the pairs of matrices that can be multiplied. For those that can be multiplied state the dimensions of the resultant matrix.

$$A\begin{bmatrix} 6 & 7 \\ -4 & 1 \end{bmatrix} \quad B\begin{bmatrix} 10 \\ 9 \\ -1 \end{bmatrix} \quad C\begin{bmatrix} -5 & 8 & 0 \\ 3 & 6 & -8 \end{bmatrix} \quad D\begin{bmatrix} 13 & 25 \\ -11 & 4 \\ 6 & 19 \end{bmatrix}$$

2x2 3x1 2x3 3x2

AC

$$\begin{bmatrix} -9 & 90 & -56 \\ 23 & -26 & -8 \end{bmatrix}$$

CB $\begin{bmatrix} 22 \\ 92 \end{bmatrix}$

CD $\begin{bmatrix} -153 & -93 \\ -75 & -53 \end{bmatrix}$

DA $\begin{bmatrix} -22 & 116 \\ -82 & -73 \\ -40 & 61 \end{bmatrix}$

DC $\begin{bmatrix} 10 & 254 & -200 \\ 67 & -64 & -32 \\ 27 & 162 & -152 \end{bmatrix}$

Is matrix multiplication commutative?

$$A\begin{bmatrix} 4 & 6 & -3 \\ 7 & 1 & 8 \end{bmatrix} \quad B\begin{bmatrix} 6 & 5 \\ -9 & 0 \\ 6 & -1 \end{bmatrix} \quad C\begin{bmatrix} 1 & 11 \\ 2 & -3 \end{bmatrix} \quad D\begin{bmatrix} -5 & 4 \\ 2 & 19 \end{bmatrix}$$

2x3 3x2

Is A•B the same as B•A?

2x2 3x3

These don't have the same dimensions. Therefore, AB is not the same as BA.

Is C•D the same as D•C?

2x2 2x2

$$\begin{bmatrix} 17 & 213 \\ -16 & -49 \end{bmatrix} \quad \begin{bmatrix} 3 & -67 \\ 40 & -53 \end{bmatrix}$$

These have the same dimensions but not the same elements. Therefore, CD is not the same as DC.

These two examples of matrix multplication show that matrix multiplication is NOT commutative.

Properties **Matrix Multiplication**

If A, B, and C are $n \times n$ matrices, then

AB is an $n \times n$ matrix.	Closure Property
$(AB)C = A(BC)$	Associative Property of Multiplication
$A(B + C) = AB + AC$ $(B + C)A = BA + CA$	Distributive Property
$OA = AO = O$, where O has the same dimensions as A.	Multiplicative Property of Zero

	Lunch 1	Lunch 2	Lunch 3
Cost per Lunch	$2.50	$1.75	$2.00
Number Sold	50	100	75

Find the total amount of money spent on all the lunches sold.

(2.50)(50) + (1.75)(100) + (2.00)(75) = $450

1. Write a 1x3 matrix(C) to represent the cost of the lunches.

$$C\begin{bmatrix} \$2.50 & \$1.75 & \$2.00 \end{bmatrix}$$

2. Write a 3x1 matrix(N) to represent the number of lunches sold.

$$N\begin{bmatrix} 50 \\ 100 \\ 75 \end{bmatrix}$$

	Lunch 1	Lunch 2	Lunch 3
Cost per Lunch	$2.50	$1.75	$2.00
Number Sold	50	100	75

Total $ spent = ($ per lunch)x(# of lunches)

$$C\begin{bmatrix} \$2.50 & \$1.75 & \$2.00 \end{bmatrix} \quad N\begin{bmatrix} 50 \\ 100 \\ 75 \end{bmatrix}$$

$1 \times 3 \cdot 3 \times 1 = 1 \times 1$

$$C \bullet N = \begin{bmatrix} \$2.50 & \$1.75 & \$2.00 \end{bmatrix} \bullet \begin{bmatrix} 50 \\ 100 \\ 75 \end{bmatrix}$$

= (2.50)(50) + (1.75)(100) + (2.00)(75)

= $450

3. Describe a procedure for using the matrices to find how much money will be collected from selling all three lunches. Make sure you use the words *row, column, and element*.

Multiply the elements in the first row of matrix C with the corresponding elements in the first column of matrix N, then add them together.

Multiplying matrices by hand:

$$A\begin{bmatrix} 6 & 7 \\ -4 & 1 \end{bmatrix} \quad B\begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

Find this product: A • B

$$A \bullet B = 2x2 \bullet 2x1 = 2x1 = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 44 \\ -18 \end{bmatrix}$$

Element a: Location: Row 1, Column 1. a = Row 1(A) x Col 1(B)

$6 \cdot 5 + 7 \cdot 2 = 44$

Element b: Location: Row 2, Column 1. b = Row 2(A) x Col 1(B)

$-4 \cdot 5 + 1 \cdot 2 = -18$