

Multiplying a Matrix by Another Matrix

But to multiply a matrix **by another matrix** we need to do the "dot product" of rows and columns ... what does that mean? Let us see with an example:

To work out the answer for the **1st row** and **1st column**:

Matrix Multiplication

Rows x Columns

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & \end{bmatrix}$$

"Dot Product"

The "Dot Product" is where we **multiply matching members**, then sum up:

$$(1, 2, 3) \cdot (7, 9, 11) = 1 \times 7 + 2 \times 9 + 3 \times 11 = 58$$

We match the 1st members (1 and 7), multiply them, likewise for the 2nd members (2 and 9) and the 3rd members (3 and 11), and finally sum them up.

Without using a graphing calculator find the product of each pair of matrices.

1. $\begin{bmatrix} 4 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 6 \end{bmatrix}$

2. $\begin{bmatrix} 7 & -1 & 5 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 6 & -1 \\ 9 & 0 \end{bmatrix}$

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$$1. \begin{bmatrix} 4 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$2 \times 2 \cdot 2 \times 2 = 2 \times 2$$

$$a = \text{Row 1} \cdot \text{Col 1}$$

$$= 4 \cdot 1 + 2 \cdot -2 = 4 + -4 = 0$$

$$b = \text{Row 1} \cdot \text{Col 2}$$

$$= 4 \cdot 0 + 2 \cdot 6 = 0 + 12 = 12$$

$$c = \text{Row 2} \cdot \text{Col 1}$$

$$= -5 \cdot 1 + 3 \cdot -2 = -5 + -6 = -11$$

$$d = \text{Row 2} \cdot \text{Col 2}$$

$$= -5 \cdot 0 + 3 \cdot 6 = 0 + 18 = 18$$

$$= \begin{bmatrix} 0 & 12 \\ -11 & 18 \end{bmatrix}$$

$$2. \begin{bmatrix} 7 & -1 & 5 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 6 & -1 \\ 9 & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$2 \times 3 \cdot 3 \times 2 = 2 \times 2$$

$$a = \text{Row 1} \cdot \text{Col 1}$$

$$= 7 \cdot 3 + -1 \cdot 6 + 5 \cdot 9 = 21 - 6 + 45 = 60$$

$$b = \text{Row 1} \cdot \text{Col 2}$$

$$= 7 \cdot -2 + -1 \cdot -1 + 5 \cdot 0 = -14 + 1 + 0 = -13$$

$$c = \text{Row 2} \cdot \text{Col 1}$$

$$= 2 \cdot 3 + 1 \cdot 6 + 4 \cdot 9 = 6 + 6 + 36 = 48$$

$$d = \text{Row 2} \cdot \text{Col 2}$$

$$= 2 \cdot -2 + 1 \cdot -1 + 4 \cdot 0 = -4 + -1 + 0 = -5$$

$$= \begin{bmatrix} 60 & -13 \\ 48 & -5 \end{bmatrix}$$