Properties	Scalar Multiplication
If A, B, and O are $m \times n$ matrices and c and d are scalars, then	
cA is an $m \times n$ matrix.	Closure Property
$(cd)A = c(dA)$	Associative Property of Multiplication
$c(A + B) = cA + cB$ $(c + d)A = cA + dA$	Distributive Property
$1 \cdot A = A$	Multiplicative Identity Property
$0 \cdot A = O$ and $cO = O$	Multiplicative Property of Zero

Enter these matrices on the graphing calculator:

$A \begin{bmatrix} 8 & 5 \\ -3 & 7 \end{bmatrix}$ $B \begin{bmatrix} 10 & 3 & 9 \\ 17 & -5 & 1 \end{bmatrix}$ $C \begin{bmatrix} 4 & 20 \\ 0 & 2 \\ -1 & 6 \end{bmatrix}$ $D \begin{bmatrix} 7 & 13 & 8 \\ -1 & 4 & 10 \end{bmatrix}$

1. Which matrices can be added? B & D

2. Which can be subtracted? B & D

3. Is matrix addition and subtraction commutative?

Yes

No, unless you change subtraction into adding the opposite.

$A \begin{bmatrix} 8 & 5 \\ -3 & 7 \end{bmatrix}$ $B \begin{bmatrix} 10 & 3 & 9 \\ 17 & -5 & 1 \end{bmatrix}$ $C \begin{bmatrix} 4 & 20 \\ 0 & 2 \\ -1 & 6 \end{bmatrix}$

$D \begin{bmatrix} 11 & -4 & -8 \end{bmatrix}$

4. Experimenting with the graphing calculator find which matrices can be multiplied.

5. Write down the ones that can be multiplied, with their dimensions, and the dimensions of the resulting matrix.

See next page for the anwers to this.

A $\begin{bmatrix} 8 & 5 \\ -3 & 7 \end{bmatrix}$ B $\begin{bmatrix} 10 & 3 & 9 \\ 17 & -5 & 1 \end{bmatrix}$ C $\begin{bmatrix} 4 & 20 \\ 0 & 2 \\ -1 & 6 \end{bmatrix}$ D $\begin{bmatrix} 11 & -4 & -8 \end{bmatrix}$

Only the following matrix multiplications are possible:

A • B = 2x3
(2x2) (2x3)

C • B = 3x3
(3x2) (2x3)

B • C = → 2x2
(2x3) (3x2)

D • C = 1x2
(1x3) (3x2)

C • A = 3x2
(3x2)(2x2)

To Add and Subtract two matrices they must have the exact same dimensions.

To multiply two matrices the second matrix must have the same number of rows as the number of columns in the first matrix.

The "middle numbers" must match in order to multiply two matrices.

(3 x 2)(2 x 3) or (2 x 3)(3 x 2)

the dimensions of the answer are the first and last numbers of the two matrices being multiplied. Or, "the outside two #'s"

(3 x 2)(2 x 3) = 3x3 or (2 x 3)(3 x 2) = 2x2

State all the pairs of matrices that can be multiplied. For those that can be multiplied state the dimensions of the resultant matrix.

A $\begin{bmatrix} 6 & 7 \\ -4 & 1 \end{bmatrix}$ B $\begin{bmatrix} 10 \\ 9 \\ -1 \end{bmatrix}$ C $\begin{bmatrix} -5 & 8 & 0 \\ 3 & 6 & -8 \end{bmatrix}$ D $\begin{bmatrix} 13 & 25 \\ -11 & 4 \\ 6 & 19 \end{bmatrix}$

2x2 3x1 2x3 3x2

A•C = 2x3
C•B = 2x1
C•D = 2x2
D•A = 3x2
D•C = 3x3

Is matrix multiplication commutative?

A $\begin{bmatrix} 4 & 6 & -3 \\ 7 & 1 & 8 \end{bmatrix}$ B $\begin{bmatrix} 6 & 5 \\ -9 & 0 \\ 6 & -1 \end{bmatrix}$ C $\begin{bmatrix} 1 & 11 \\ 2 & -3 \end{bmatrix}$ D $\begin{bmatrix} -5 & 4 \\ 2 & 19 \end{bmatrix}$

Is A•B the same as B•A?

2x3 · 3x2
2x2

3x2 · 2x3
3x3

These will have different dimensions, therefore, AB is not the same as BA

Is C•D the same as D• C?

2x2 · 2x2
2x2
$\begin{bmatrix} 17 & 213 \\ -16 & -49 \end{bmatrix}$

2x2 · 2x2
2x2
$\begin{bmatrix} 3 & -67 \\ 40 & -35 \end{bmatrix}$

Even though these have the same dimensions they have different elements, therefore, CD is not the same as DC

Thus, matrix multiplication is NOT Commutative.

Properties **Matrix Multiplication**

If A, B, and C are $n \times n$ matrices, then

AB is an $n \times n$ matrix.	Closure Property
$(AB)C = A(BC)$	Associative Property of Multiplication
$A(B + C) = AB + AC$ $(B + C)A = BA + CA$	Distributive Property
$OA = AO = O$, where O has the same dimensions as A.	Multiplicative Property of Zero

	Lunch 1	Lunch 2	Lunch 3
Cost per Lunch	$2.50	$1.75	$2.00
Number Sold	50	100	75

1. Write a 1x3 matrix(C) to represent the cost of the lunches. $C \begin{bmatrix} 2.50 & 1.75 & 2.00 \end{bmatrix}$

2. Write a 3x1 matrix(N) to represent the number of lunches sold. $N \begin{bmatrix} 50 \\ 100 \\ 75 \end{bmatrix}$

3. Describe a procedure for using the matrices to find how much money will be collected from selling all three lunches. Make sure you use the words *row, column, and element*.

$(2.50)(50) + (1.75)(100) + (2)(75)$

Multiply the elements in the 1st matrix by the corresponding elements of the 2nd matrix then add them together.

	Lunch 1	Lunch 2	Lunch 3
Cost per Lunch	$2.50	$1.75	$2.00
Number Sold	50	100	75

Total amount of $ collected:

C • N = $450

$C\begin{bmatrix} \$2.50 & \$1.75 & \$2.00 \end{bmatrix} \bullet N\begin{bmatrix} 50 \\ 100 \\ 75 \end{bmatrix}$

= (2.50)(50)+(1.75)(100)+(2.00)(75) = $450

Definition **Matrix Multiplication**

To find element c_{ij} of the product matrix AB, multiply each element in the ith row of A by the corresponding element in the jth column of B, and then add.

Remember this phrase: "Row times Column"

Multiplying matrices by hand:

$$A\begin{bmatrix} 6 & 7 \\ -4 & 1 \end{bmatrix} \quad B\begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

Find this product: $A \bullet B$

$$A \bullet B = 2x2 \bullet 2x1 = 2x1 = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 44 \\ -18 \end{bmatrix}$$

Element a: Location: Row 1, Column 1.

a = Row 1(A) x Col 1(B)

$6 \cdot 5 + 7 \cdot 2 = 30 + 14 = 44$

Element b: Location: Row 2, Column 1.

b = Row 2(A) x Col 1(B)

$-4 \cdot 5 + 1 \cdot 2 = -18$