Matrix Division:

Dividing

And what about division? Well we **don't** actually divide matrices, we do it this way:

$$A/B = A \times (1/B) = A \times B^{-1}$$

where $\mathbf{B}^{-1}$ means the "inverse" of B.

So we don't divide, instead we **multiply by an inverse**.

Definition **Multiplicative Identity Matrix**

For an $n \times n$ square matrix, the **multiplicative identity matrix** is an $n \times n$ square matrix I, or I_n, with 1's along the main diagonal and 0's elsewhere.

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, and so forth

Use A $\begin{bmatrix} 6 & -9 \\ 4 & 5 \end{bmatrix}$ to show that I_2 $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity matrix

$$\begin{bmatrix} 6 & -9 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -9 \\ 4 & 5 \end{bmatrix}$$

this shows that $A \cdot I_2 = A$

therefore, I_2 must be the Identity Matrix

Find each.

a. $5 \cdot 5^{-1} = 1$

b. $-12 \cdot (-12)^{-1} = 1$

Since the product in each case is 1, these numbers are inverses of each other.

5^{-1} and $(-12)^{-1}$ are called

Multiplicative Inverses of 5 and -12, respectively.

Solve this equation without using division or fractions.

$$6^{-1} \cdot 6x = 45 \cdot 6^{-1}$$

$$\boxed{x = 7.5}$$

Definition **Multiplicative Inverse of a Matrix**

If A and X are $n \times n$ matrices, and $AX = XA = I$, then X is the multiplicative inverse of A, written A^{-1}.

$$AA^{-1} = A^{-1}A = I$$

this state that if you find the product of two matrices, both ways, and get the Identity Matrix as a result, then one matrix is the multiplicative inverse of the other matrix.

Show that matrices A and B are inverses.

$$A \begin{bmatrix} 4 & -2 \\ -9 & 5 \end{bmatrix} \qquad B \begin{bmatrix} 2.5 & 1 \\ 4.5 & 2 \end{bmatrix}$$

$$AB = BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

To find the inverse of a matrix takes several steps.

We first must find the Determinant of the matrix.

Definition **Determinant of a 2 × 2 Matrix**

The **determinant** of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - bc$.

Symbols for the determinant of a matrix

$\downarrow$ $\qquad$ $\downarrow$

$\det A \qquad \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

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Find the determinant of matrix A:

$$A \quad \begin{array}{cc} a & b \\ \begin{bmatrix} 6 & 10 \\ 5 & 8 \end{bmatrix} & \\ c & d \end{array}$$

$$\det A = 6 \cdot 8 - 10 \cdot 5$$
$$= 48 - 50$$
$$\det A = -2$$

A $\begin{bmatrix} 6 & 10 \\ 5 & 8 \end{bmatrix}$ det A= -2

The Determinant of a matrix is used to find its Inverse:

Property	**Inverse of a 2 × 2 Matrix**

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If $\det A = 0$, then A has no inverse.

If $\det A \neq 0$, then $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Find $A^{-1} = \frac{1}{-2}\begin{bmatrix} 8 & -10 \\ -5 & 6 \end{bmatrix} = \boxed{\begin{bmatrix} -4 & 5 \\ 2.5 & -3 \end{bmatrix}}$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \longrightarrow A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

A matrix and it's inverse have the same dimensions

Not every matrix has an inverse because
the Determinant could be zero and this would make

$\frac{1}{\det A}$ undefined.

A $\begin{bmatrix} 6 & 10 \\ 5 & 8 \end{bmatrix}$ $A^{-1}\begin{bmatrix} -4 & 5 \\ 2.5 & -3 \end{bmatrix}$

Confirm that these matrices are inverses.

$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{-1} \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

since the product of these two
matrices gives the Identity Matrix,
both ways, they must be inverses.

Find B^{-1}

B $\begin{bmatrix} -2 & -3 \\ 8 & 14 \end{bmatrix}$

det B:

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - bc$.

$= -4$

B^{-1}

$\frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\frac{1}{-4}\begin{bmatrix} 14 & 3 \\ -8 & -2 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} -\frac{7}{2} & -\frac{3}{4} \\ 2 & \frac{1}{2} \end{bmatrix}$