What's going to be important for this next topic is:

Whether the Leading Coefficient is Positive or Negative

Whether the degree is Even or Odd

$$y=(3x-8)^4(7-2x)^3(12x-6)(11-7x)^4(8-x)^7(9+8x)^5$$

$$+ \cdot - \cdot + \cdot + \cdot - \cdot +$$

DEG: EVEN 4+3+1+4+7+5

LC: POS

End Behavior of polynomial graphs.

Where are the ends of a graph found?

At the far left and far right

Where x is a very big positive # and a very big negative #

When you are asked to describe the end behavior of a graph you are really asked to describe what the value of Y is doing at the very far left and right of the graph.

End-Behavior:

The behavior of the graph on the far left and the far right.

How the value of the function (y) changes as x becomes
larger negative LEFT END $x \rightarrow -\infty$
and
larger positive RIGHT END. $x \rightarrow \infty$

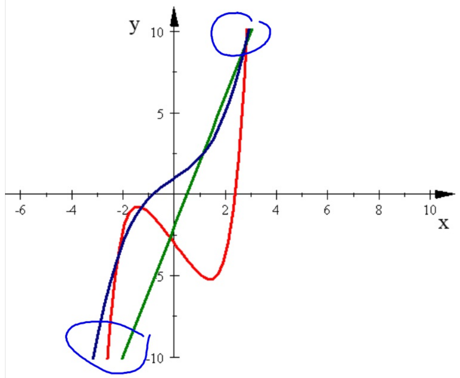
At the ends of a graph Y will be doing only one of three things:

- Increasing $y \longrightarrow \infty$
- Decreasing $y \longrightarrow -\infty$
- Flattens out and approaches a constant

Graph all three of these in a Standard Window:

$$Y_1 = 4x - 2$$
$$Y_2 = 0.25x^3 + x + 1$$
$$Y_3 = 0.1x^5 - 2x - 3$$

What do the graphs have in common?



$$Y_1 = 4x - 2$$
$$Y_2 = 0.25x^3 + x + 1$$
$$Y_3 = 0.1x^5 - 2x - 3$$

Same end behavior

LEFT as $x \rightarrow -\infty$ $y \rightarrow -\infty$

RIGHT as $x \rightarrow \infty$ $y \rightarrow \infty$

Or down on the left and up on the right

What do the equations have in common?

	Degree	Lead Coeff
$Y_1 = 4x - 2$	1	+
$Y_2 = 0.25x^3 + x + 1$	3	+
$Y_3 = 0.1x^5 - 2x - 3$	5	+
	ODD	pos

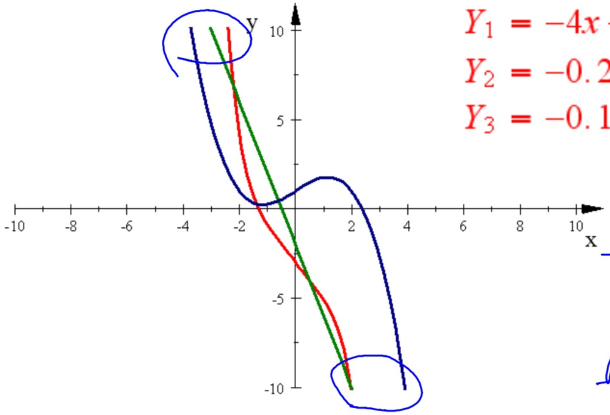
$Y_1 = 4x - 2$

$Y_2 = 0.25x^3 + x + 1$

$Y_3 = 0.1x^5 - 2x - 3$

What would happen if they all had a negative leading coefficient?

X-AXIS Reflection
upside down



$Y_1 = -4x - 2$

$Y_2 = -0.25x^3 + x + 1$

$Y_3 = -0.1x^5 - 2x - 3$

Left
as $x \to -\infty$
$y \to \infty$

Right
as $x \to \infty$ $y \to -\infty$

or up on the left and down on the right

Odd Functions: Largest exponent is ODD when expanded
This is called the degree of the function.

Positive Leading Coefficient:
Moves from the third quadrant to the first quadrant.
Like a line with a Positive slope
Down and to the left-----Up and to the right

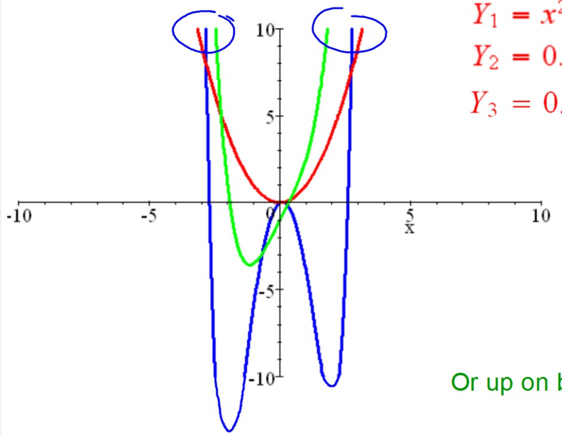
Negative Leading Coefficient:
Moves from the second quadrant to the fourth quadrant.
Like a line with a Negative slope
Up and to the left-----Down and to the right

This is called End Behavior

Graph all three of these in a Standard Window:

$$Y_1 = x^2$$
$$Y_2 = 0.5x^4 + 3x - 1$$
$$Y_3 = 0.1x^6 - 5x^2 + x$$

What do the graphs have in common?



$Y_1 = x^2$
$Y_2 = 0.5x^4 + 3x - 1$
$Y_3 = 0.1x^6 - 5x^2 + x$

Same end behavior

as $x \to \pm\infty$
$y \to \infty$

Or up on both left and right sides

What do the equations have in common?

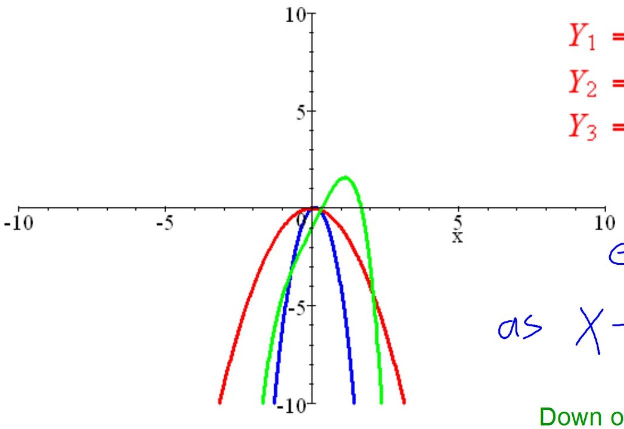
	Degree	Lead Coeff
$Y_1 = x^2$	2	+
$Y_2 = 0.5x^4 + 3x - 1$	4	+
$Y_3 = 0.1x^6 - 5x^2 + x$	6	+
	↓ even	↓ pos

$Y_1 = -x^2$
$Y_2 = -0.5x^4 + 3x - 1$
$Y_3 = -0.1x^6 - 5x^2 + x$

What would happen if they all had a negative leading coefficient?

X-axis Reflection

or Upside Down



$Y_1 = -x^2$

$Y_2 = -0.5x^4 + 3x - 1$

$Y_3 = -0.1x^6 - 5x^2 + x$

end behavior

as $x \to \pm\infty$

$y \to -\infty$

Down on both left and right sides

Even Functions: Largest exponent is EVEN when expanded

This is called the degree of the function.

Positive Leading Coefficient:

Moves from the second quadrant to the first quadrant.

Like a parabola with a>0

Up and to the left ---- Up and to the right

Negative Leading Coefficient:

Moves from the third quadrant to the fourth quadrant.

Like a parabola with a<0

Down and to the left ---- Down and to the right

You know the end behavior of every Polynomial
if you know the end behavior of

Either

A Line (ODD)

OR

A Parabola (EVEN)

END BEHAVIOR

EVEN Functions:

Positive Leading Coefficient:	Negative Leading Coefficient:	
(↖ , ↗)	(↙ , ↘)	Our Book
as $x \to -\infty, y \to \infty$	as $x \to -\infty, y \to -\infty$	Other Authors
as $x \to \infty, y \to \infty$	as $x \to \infty, y \to -\infty$	

END BEHAVIOR

ODD Functions:

Positive Leading Coefficient:	Negative Leading Coefficient:
$(\swarrow, \nearrow)$	$(\nwarrow, \searrow)$
as $x \to -\infty$, $y \to -\infty$	as $x \to -\infty$, $y \to \infty$
as $x \to \infty$, $y \to \infty$	as $x \to \infty$, $y \to -\infty$

Our Book

Other Authors

State the end behavior of each polynomial.

One way to state end behavior

1. $y = 4x^3 - 6x^2 + 11x - 93$
   POS ODD $(\swarrow, \nearrow)$
2. $y = 5x(x+2)(x-7)^2$
   POS EVEN $(\nwarrow, \nearrow)$
3. $f(x) = 9x + 6x^2 - x^3 + 13$
   NEG ODD $(\nwarrow, \searrow)$
4. $y = (9x-7)(4-x)$
   NEG EVEN $(\swarrow, \searrow)$

State the end behavior of each polynomial.

2nd way to state end behavior

1. $y = \underline{4}x^3 - 6x^2 + 11x - 93$
   POS ODD
   as $x \to -\infty$ $y \to -\infty$
   as $x \to \infty$ $y \to \infty$
2. $y = 5x(x+2)(x-7)^2$
   POS EVEN
   as $x \to \pm\infty$ $y \to \infty$
3. $f(x) = 9x + 6x^2 - x^3 + 13$
   NEG ODD
   as $x \to -\infty$ $y \to \infty$
   as $x \to \infty$ $y \to -\infty$
4. $y = (9x-7)(4-x)$
   NEG EVEN
   as $x \to \pm\infty$ $y \to -\infty$

You can now finish Hwk #25:

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Problems 1-10