

The graph of an equation is with two variables is...

A picture of all the ordered pairs that are solutions to the equation.

The graph of  $y = 2x - 1$  is

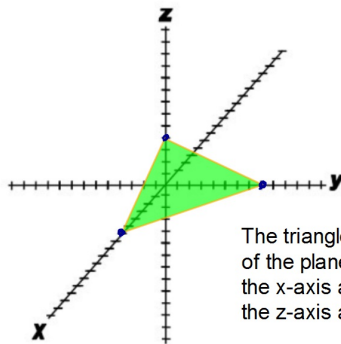
a line on the x-y plane that represents all the points that solutions to the equation.

The graph of an equation with three variables is...

A picture of all the ordered triples in space that are solutions to the equation.

This will create a PLANE - flat surface to extends forever

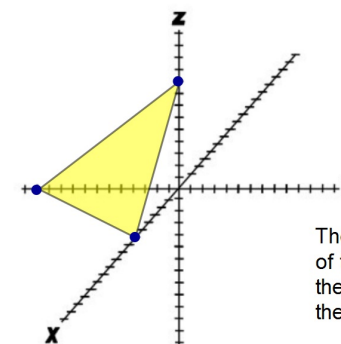
Graph this equation:  $4x + 3y + 6z = 24$



$$\begin{aligned}x\text{-int} &= 6 \\y\text{-int} &= 8 \\z\text{-int} &= 4\end{aligned}$$

The triangle is a representation of the plane in space that passes through the x-axis at 6, the y-axis at 8 and the z-axis at 4.

Graph this equation:  $24x - 12y + 16z = 144$



$$\begin{aligned}x\text{-int} &= \frac{144}{24} = 6 \\y\text{-int} &= \frac{144}{-12} = -12 \\z\text{-int} &= \frac{144}{16} = 9\end{aligned}$$

The triangle is a representation of the plane in space that passes through the x-axis at 6, the y-axis at -12 and the z-axis at 9.

A system of equations in 2 variables is:

2 lines on the x-y plane.

Solutions to systems of equations in 2 variables:

One Solution - Lines intersect once

No Solution - Lines are parallel

Many Solutions - The two equations are the same line

Systems of equations in 3 variables is ...

3 planes in Space

Systems of equations in three variables.

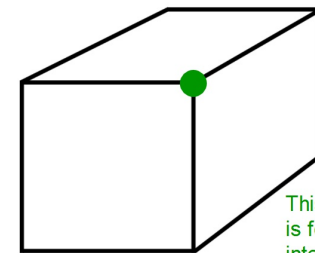
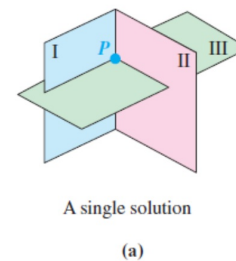
How many solutions are possible?

One Solution

No Solution

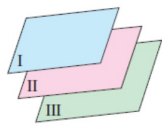
Many Solutions

One Solution:



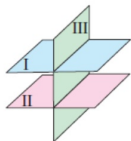
This corner of the box is formed by 3 planes intersecting, the front, top, and right side.

No solution.

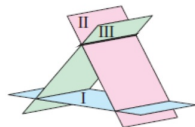


No points in common

(d)



No points in common  
to all three planes -  
at the same time



No points in common  
to all three planes -  
at the same time

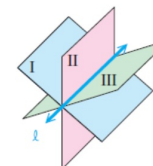
Many Solutions:



All points in common

(c)

All three equations  
are the **SAME** plane.



Points of a line in common

(b)

Just like 3 pages  
in a book all meet  
along a line - and lines  
have infinite # of points

Solving a system of equations in three variables.

You can use:

- Substitution
- Elimination
- Combination of Substitution and Elimination.

Essentially you start by turning the 3 equations in 3 variables  
into two equations with two variables (you eliminate one of them).

Then you can use Elimination or Substitution to  
solve for those two variables.

Once you know two of the three variables you can  
substitute them into an equation and find the 3rd variable.

One of the main rules when solving a system of three equations is that you must use each equation at least once.

Solve this system of equations using Algebra.

$$4x + 2y - z = 33$$

$$y = 3z - 1$$

$$3x + 7z = 36$$

Use substitution with the first two equations:

$$4x + 2(3z - 1) - z = 33$$

$$4x + 6z - 2 - z = 33$$

$$4x + 5z = 35$$

Use this result and the third equation as a system to solve for x and z.

$$\begin{array}{r} 3(4x + 5z = 35) \quad 12x + 15z = 105 \\ 4(3x + 7z = 36) \quad -12x + 28z = 144 \\ \hline -13z = -39 \end{array}$$

$$z = 3$$

Use this value of z to find both x and y

$$\begin{aligned} 3x + 7(3) &= 36 \\ 3x + 21 &= 36 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

Solution  
(5, 8, 3)

$$\begin{aligned} y &= 3(3) - 1 \\ y &= 8 \end{aligned}$$