

Solving a system of equations in three variables.

You can use:

- Substitution
- Elimination
- Combination of Substitution and Elimination.

One of the main rules when
solving a system of three equations
is that you must
use each equation at least once.

Solve starting with Elimination.

1. $2x - 4y + 3z = 11$
2. $7x + 3y - 5z = -39$
3. $3x + 8y + 4z = 14$

Eliminate y using EQ's 1 and 2.

$$\begin{array}{r} 3(2x - 4y + 3z = 11) \\ 4(7x + 3y - 5z = -39) \\ \hline 6x - 12y + 9z = 33 \\ 28x + 12y - 20z = -156 \\ \hline \textcircled{A} \quad 34x - 11z = -123 \end{array}$$

1. $2x - 4y + 3z = 11$
2. $7x + 3y - 5z = -39$
3. $3x + 8y + 4z = 14$

Eliminate y using EQ's 1 and 3.

$$\begin{array}{r} 2(2x - 4y + 3z = 11) \rightarrow 4x - 8y + 6z = 22 \\ 3x + 8y + 4z = 14 \rightarrow + 3x + 8y + 4z = 14 \\ \hline \textcircled{B} \quad 7x + 10z = 36 \end{array}$$

Use Equations A and B and solve for x & y:

$$\begin{array}{rcl} \textcircled{A} & 10(34x - 11z = -123) & 340x - 110z = -1230 \\ \textcircled{B} & 11(7x + 10z = 36) & + \quad 77x + 110z = 396 \\ \hline & & 417x = -834 \\ & & \frac{417x}{417} = \frac{-834}{417} \\ & & \boxed{x = -2} \end{array}$$

Now use either Eq A or B to find z:

$$\begin{array}{rcl} \textcircled{B} & 7(-2) + 10z = 36 & \\ & -14 + 10z = 36 & \\ & +14 & +14 \\ & \frac{10z = 50}{10} & \frac{50}{10} \\ & \boxed{z = 5} \end{array}$$

Solving a system of equations in three variables starting with SUBSTITUTION:

1. $5x + 3y + 3z = 51$
2. $4x - 2y + z = 22$
3. $x + 5y - 2z = 13$

1. Solve one of the equations for one of its variables.

you could solve the second equation for z:

$$z = 22 + 2y - 4x$$

Now that you know that $x = -2$ and $z = 5$ use one of the first three equations to find y.

1. $2x - 4y + 3z = 11$
2. $7x + 3y - 5z = -39$
3. $3x + 8y + 4z = 14 \rightarrow$

Solution to this system is: $(-2, 0, 5)$

$$\begin{array}{l} 3(-2) + 8y + 4(5) = 14 \\ -6 + 8y + 20 = 14 \\ 8y + 14 = 14 \\ 8y = 0 \\ \boxed{y = 0} \end{array}$$

1. $5x + 3y + 3z = 51$
2. $4x - 2y + z = 22$
3. $x + 5y - 2z = 13$

2. Substitute this quantity into both of the other two equations creating a system of equations with just two variables.

Into Eq 1:

$$\begin{array}{l} 5x + 3y + 3(22 + 2y - 4x) = 51 \\ 5x + 3y + 66 + 6y - 12x = 51 \\ -7x + 9y = -15 \end{array}$$

Into Eq 2:

$$\begin{array}{l} x + 5y - 2(22 + 2y - 4x) = 13 \\ x + 5y - 44 - 4y + 8x = 13 \\ 9x + y = 57 \end{array}$$

3. Solve this resulting system of equations for its two variables.

$$\begin{array}{l} -7x + 9y = -15 \rightarrow \\ \Rightarrow (9x + y = 57) \rightarrow \end{array} \quad \begin{array}{r} -7x + 9y = -15 \\ 81x + 9y = 513 \\ \hline -88x = -528 \\ \boxed{x = 6} \end{array}$$

solve for y using this second equation and the value of x you just found:

$$\begin{array}{l} 9(6) + y = 57 \\ 54 + y = 57 \\ \boxed{y = 3} \end{array}$$

1. $5x + 3y + 3z = 51$

2. $4x - 2y + z = 22$

3. $x + 5y - 2z = 13$

4. Substitute these two values into one of the original equations and solve for z.

$$x = 6 \quad y = 3$$

using Eq 2:

$$4(6) - 2(3) + z = 22$$

$$24 - 6 + z = 22$$

$$18 + z = 22$$

$$\boxed{z = 4}$$

Solution to this system is:
 $(6, 3, 4)$

You can now finish Hwk #20:
Sec 3-6

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