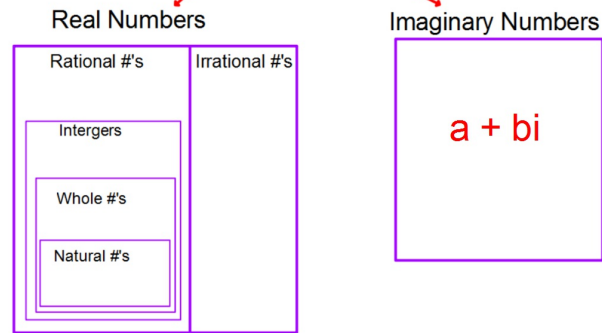


Sec 5-6

Complex Numbers



Imaginary Numbers:

$$\sqrt{-1} = i$$

i is called the imaginary unit.

Complex Numbers:

any number that can be written
in the form: $a + bi$ (a and b can be any real #)

Standard Form
of a Complex
Number

Real
Part

Imaginary
Part

Real Number: when $b=0$

Imaginary Number: when $b \neq 0$ (a may or may not be zero)

Examples of Imaginary #'s: $10 - 7i$ or $13i$

The terms Complex Number and Imaginary Number
are quite often used interchangeably.

Even though they are NOT the same thing!

Simplify each.

a. $\sqrt{-196}$
 $= \sqrt{196 \cdot -1}$
 $= \sqrt{196} \cdot \sqrt{-1}$
 $= 14i$

b. $\sqrt{-288}$
 $= \sqrt{144 \cdot 2 \cdot -1}$
 $= \sqrt{144} \cdot \sqrt{-1} \cdot \sqrt{2}$
 $= 12i\sqrt{2}$

Write each as a Complex Number in Standard Form

a. $-9 + \sqrt{-24}$
 $\sqrt{24} = \sqrt{4 \cdot 6} = 2\sqrt{6}$
 $-9 + 2i\sqrt{6}$

b. $-\sqrt{-100} + 11$
 $11 - 10i$

Simplify each expression. Write complex numbers in Standard Form.

a. $(7 + \sqrt{-1}) - (-2 + \sqrt{-81})$
 $(7 + i) - (-2 + 9i)$
 $9 - 8i$

b. $(4 - \sqrt{25}) + (-12 - \sqrt{-64})$
 $(4 - 5) + (-12 - 8i)$
 $-1 - 12 - 8i$
 $-13 - 8i$

$i = \sqrt{-1}$

$i^2 = \sqrt{-1} \cdot \sqrt{-1}$ or $(\sqrt{-1})^2 = -1$

Simplify each:

1. $4i(3 + 6i)$
 $12i + 24i^2$
 $12i + 24(-1)$
 $12i - 24$
 $-24 + 12i$

2. $(2 + 3i)(1 - 5i)$

	2	+3i
1	2	+3i
-5i	-10i	-15i^2 = 15

$17 - 7i$

Simplify:

a. $(1 + 5i)^2$

$$\begin{array}{r} 1 + 10i + 25i^2 \\ \quad \quad \quad -25 \\ \hline -24 + 10i \end{array}$$

b. $(4 + 2i)(4 - 2i)$

$$\begin{array}{r} 16 - 4i^2 \\ \quad \quad \quad 16 + 4 \\ \hline 20 \end{array}$$

$(x + 5)^2$ is never just 2 terms!!!

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$\begin{aligned} (x + 5)^2 &= (x)^2 + 2(1)(5)x + (5)^2 \\ &= x^2 + 10x + 25 \end{aligned}$$

However.....

$$\begin{aligned} (1 + 5i)^2 &= (1)^2 + 2(1)(5i) + (5i)^2 \\ &= -24 + 10i \end{aligned}$$

When you square a complex number you get
another complex number.

$$(3 - 4i)^2 =$$

$$\begin{array}{r} 9 - 24i + 16i^2 \\ \quad \quad \quad -16 \\ \hline -7 - 24i \end{array}$$

When dealing with Real Numbers only:

$(x + 5)(3x + 2)$ is a Trinomial

When dealing with Imaginary Numbers only:

$(5 + i)(2 + 3i)$ is a Binomial

(another imaginary number)

$$(2x - 3)(2x + 3) =$$

$$(2x)^2 - (3)^2$$

$$4x^2 - 9$$

$$(2 - 3i)(2 + 3i) =$$

$$(2)^2 - (3i)^2$$

$$4 - 9i^2$$

$$4 + 9 = 13$$

When a and b are REAL #'s

$$(a + b)(a - b) = a^2 - b^2$$

$$(4 + 2i)(4 - 2i)$$

With Imaginary Numbers:

$$(a + bi)(a - bi) = a^2 + b^2$$

Factors such as $(a + b)$ and $(a - b)$ are called **CONJUGATES**

Conjugate

The conjugate is where we **change the sign in the middle** of two terms like this:

$$\begin{array}{l} 3x + 1 \\ \downarrow \\ \text{Conjugate: } 3x - 1 \end{array}$$

Complex Conjugates: $a + bi$ and $a - bi$

$$(7 + 4i)(7 - 4i) =$$

$$49 + 16 = 65$$

The product of complex conjugates is always
a constant

Simplify each.

1. $(9 - 5i)^2$

$$\begin{array}{r} 81 - 90i + 25i^2 \\ \hline 56 - 90i \end{array}$$

(Note: A bracket under 25i^2 points to -25, and the final result 56 - 90i is circled in green.)

2. $(6 - 3i)(6 + 3i)$

$$36 + 9 = 45$$

(Note: The result 45 is circled in green.)

Hwk #22: Sec 5-6

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Due tomorrow

Problems 3-5, 13, 14, 34-36, 40, 57, 59