Solving a system of Linear Equations using matrices.

$$A\begin{bmatrix} 5 & 6 & -1 \\ 0 & -4 & 8 \end{bmatrix} \qquad B\begin{bmatrix} -9 & -1 & 2 \\ 7 & 3 & 0 \end{bmatrix} \qquad C\begin{bmatrix} 4 & -3 \\ 2 & 10 \\ -6 & 5 \end{bmatrix}$$

Which two matrices can be:

1. Added A and B: either A+B or B+A
2. Subtracted A and B: either A-B or B-A
3. Multiplied A and C or B and C
4. Divided You can't divide matrices!

To Add and Subtract two matrices they must have the exact same dimensions.

To multiply two matrices
the second matrix must have the same number of rows as the number of columns in the first matrix Their middle numbers must match:

A*C = 3 x 2 * 2 x 3 or C*B = 2 x 3 * 3 x 2

the dimensions of the answer are the first and last numbers of the two matrices being multiplied.

The dimensions of the answer matrix when you multiply two matrices:

C*B = 3 x 2 * 2 x 3 = 3 x 3

You CAN'T do matrix division. However.....

Solve this equation without dividing.

$2x = 10$

multiply both sides by the reciprocal of the coefficient

$\frac{1}{2} \cdot 2x = 10 \cdot \frac{1}{2}$

multiply both sides by the inverse of the coefficient

$2^{-1} \cdot 2x = 10 \cdot 2^{-1}$

Matrix Equation

$$A \begin{bmatrix} X \\ Y \end{bmatrix} = B$$

To solve for $\begin{bmatrix} X \\ Y \end{bmatrix}$

you would normally

divide by matrix A.

However, you can't divide matrices!!

Therefore , we multiply by the inverse of matrix A. But, whichway?

$$\begin{bmatrix} X \\ Y \end{bmatrix} = B \cdot A^{-1} \quad \text{OR} \quad \begin{bmatrix} X \\ Y \end{bmatrix} = A^{-1} \cdot B$$

$4x + 3y = 23$

$2x - 4y = 6$

Dimensions of matrix A (coefficient matrix) 2 x 2

Dimensions of matrix A^{-1} 2 x 2

Dimensions of matrix B (constant matrix) 2 x 1

$A^{-1} \bullet B$

$2 \times 2 \bullet 2 \times 1$

because these match up you CAN multiply these

or

$B \bullet A^{-1}$

$2 \times 1 \bullet 2 \times 2$

Since these don't match up you CAN'T multiply these

You will see this Error message

DIM MISMATCH

When solving systems of equations with matrices you always find the solution this way:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = A^{-1} \cdot B$$

Solve this system of equations:

$4x - 10y = 18$

$6x - 15y = 27$

when you try to solve this system of equations using matrices you get the following error message: ERR:SINGULAR MAT

This means that you can't solve this system of equations with matrices due to the fact it represents either two parallel lines (No Sol) or to lines that are actually the same (Many Sol). To determine which answer is correct you need to change these equations to slope-intercept form to find the slopes and y-intercepts in order to determine if they are parallel or the same line.

Don't have a graphing calculator to solve a system of equations with matrices?

- Borrow one
- Use the internet → On my Blog click on the "Helpful Math Links" seen on the bar across the middle.

  Scroll down until you see Online Matrix Calculator Links Choosing either one will take you to a web page that will help you solve a system of equations with matrices.

  Or you could search the internet for other ones.

How would you solve this system of equations using matrices?

$y = 4x + 9$

$y = 21$

$AX + BY = C$

$$A = \begin{bmatrix} -4 & 1 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 9 \\ 21 \end{bmatrix}$$

$[A]^{-1}[B] =$ (3,21)

You can now finish Hwk #11 Sec 4-7

Page 217 Due tomorrow

Problems 7-11

Write out Matrices A and B then give the solution as an ordered pair.

Sec 3-5: Graphs in 3 Dimensions

and

Sec 3-6: Systems with Three Variables

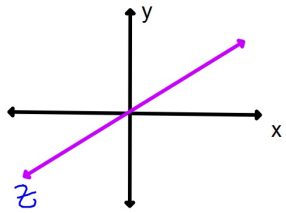
(5,-2) is called an ordered pair

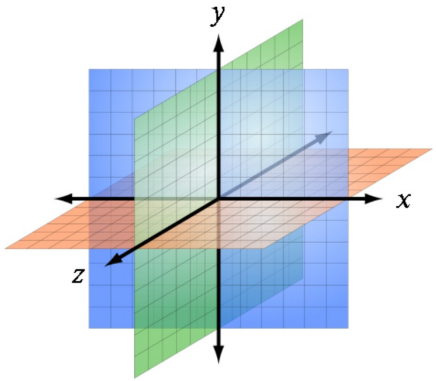
x, y

What do you think (1, -9, 3) is called? An ordered triple

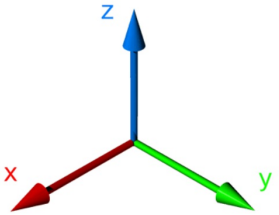
x, y, z

Plotting an ordered triple.

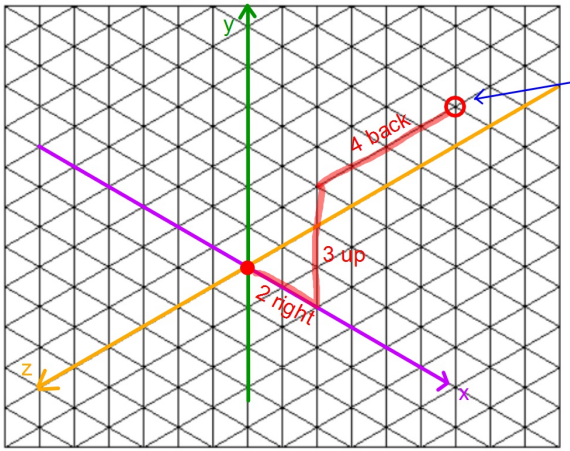




Our Book and other sources have the axes in different locations.



Isometric Graph Paper

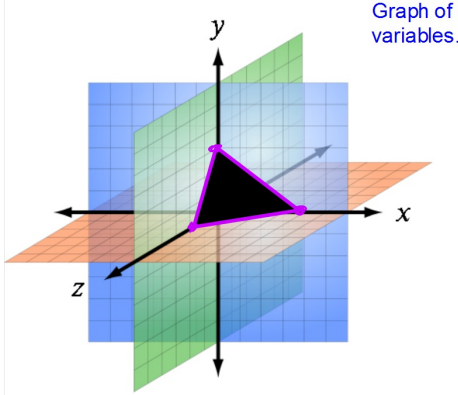


PLOT:

(2, 3, -4)

pick a point to call the origin and draw the three axes.

start at the orgin,
move 2 right (pos x),
3 up (pos y),
then 4 back (neg z).



Graph of an equation in three variables.

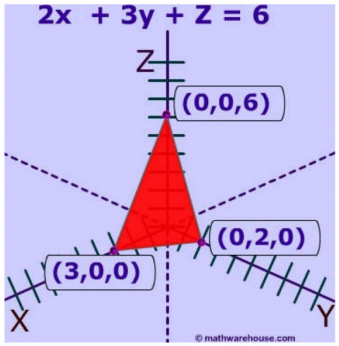
A Plane

6x + 8y + 12z = 24

x-int= $\frac{24}{6} = 4$ make y & z = 0

y-int= $\frac{24}{8} = 3$ make x & z = 0

z-int= $\frac{24}{12} = 2$ make x & y = 0



Systems of equations in three variables. 3 planes in space.

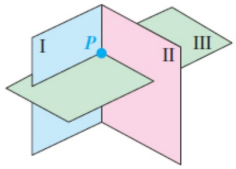
How many solutions are possible?

one

none

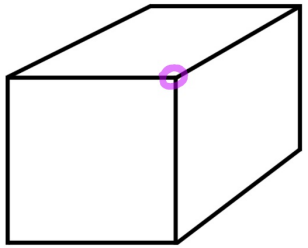
many

One Solution:



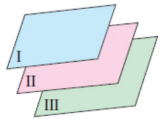
A single solution

(a)



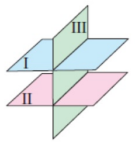
the corner of a box is where three planes intersect - top, front, and right side

No solution.



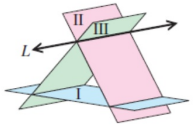
No points in common

(d)



No points in common

(e)



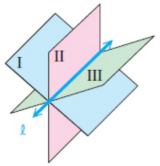
No points in common

(f)

Many Solutions:

All points in common

(c)



Points of a line in common

(b)

How would you solve this system of equations. Use matrices!

$$x + 3y + 7z = 43$$
$$4x - 3y + z = 19$$
$$x + 5y - 2z = 13$$

$$\begin{bmatrix} 1 & 3 & 7 \\ 4 & -3 & 1 \\ 1 & 5 & -2 \end{bmatrix} \quad \begin{bmatrix} 43 \\ 19 \\ 13 \end{bmatrix}$$

$$[A]^{-1}[B] = \begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix}$$

$$(6, 3, 4)$$

Solve this system of equations.

$2x - y + 3z = 21.5$

$3x - 4z = -22$

$7y + 2z = 38.5$

$[A]^{-1}[B] = (2, 3.5, 7)$

You can now finish Hwk #12 Sec 3-6

Due Wednesday

Page 158 Problems 26, 27, 30, 31
AND
Page 218 Problem 30

Write out Matrices A and B then give the solution as an ordered triple.