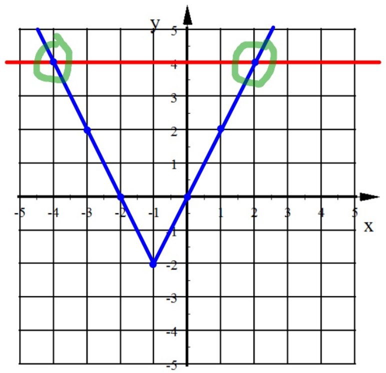
Section 1-5

Solving Absolute Value
Equations and Inequalities

The solution to every Absolute Value Equation or Inequality
ALWAYS has TWO parts.



1. Graph $y = 2|x + 1| - 2$

2. Graph $y = 4$

3. What part of this graph shows the solution to this equation:

$$2|x + 1| - 2 = 4$$

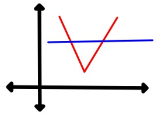
Where the graphs intersect

Therefore, the solutions are the x-coordinates of the points of intersection.

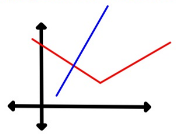
Solutions are: $x = -4, 2$

When you graph an Absolute Value "V" and a line together, how many solutions could there be?

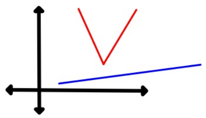
Two (2 points of intersection)



One (2 point of intersection)



None (no point of intersection)



Solve .

$5|2x - 7| - 13 = 9$

$\quad +13 \quad +13$

$\frac{5|2x-7|}{5} = \frac{22}{5}$

$|2x-7| = 4.4$

$2x-7 = -4.4 \qquad\qquad 2x-7 = 4.4$

$\;+7 \quad\; +7 \qquad\qquad +7 \quad +7$

$\frac{2x}{2} = \frac{2.6}{2} \qquad\qquad \frac{2x}{2} = \frac{11.4}{2}$

$x = 1.3 \qquad\qquad x = 5.7$

$\boxed{x = 1.3, 5.7}$

Solve this equation: $|x - 2| = 2x + 5$

to the left of zero - opposite of what is on the rt.

$x-2 = -(2x+5)$

$x-2 = -2x-5$

$+2x \qquad +2x$

$3x-2 = -5$

$\;+2 \quad +2$

$\frac{3x}{3} = \frac{-3}{3}$

$\boxed{x = -1}$

to the right of zero - POS

$x-2 = 2x+5$

$-x \qquad -x$

$-2 = x+5$

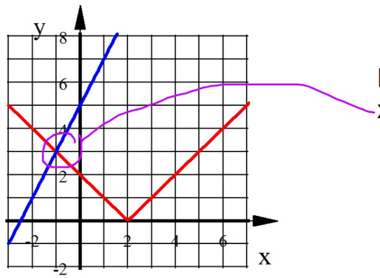
$-5 \qquad -5$

$\boxed{\cancel{x = -7}}$

When you test each solution we find that only x = -1 is actually a solution.

x=7 doesn't make the original inquality true.

If you solved this equation $|x - 2| = 2x + 5$ by graphing you would see the following graph:



From the graph you can see that x = -1 is the only solution.

Definition	Extraneous Solution

An **extraneous solution** is a solution of an equation derived from an original equation that is not a solution of the original equation.

Solve this equation:

$|2x - 1| = x + 1$

$-(x+1) \qquad 0 \qquad x+1$

$2x-1 = -(x+1)$

$2x-1 = -x-1$

$\quad +1 \qquad +1$

$2x = -x$

$+x \quad +x$

$\frac{3x}{3} = \frac{0}{3}$

$x=0$

$2x-1 = x+1$

$2x-1 = x+1$

$-x \qquad -x$

$x-1=1$

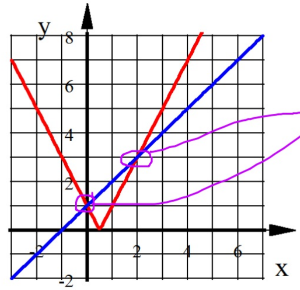
$\quad +1 \quad +1$

$x=2$

both are solutions

$\boxed{x = 0, 2}$

If you solved this equation $|2x - 1| = x + 1$ by graphing you would see the following graph:



Both x=0 and x=2 are solutions

$|x - 6| = 2x + 3$

$-(2x+3) \qquad 0 \qquad 2x+3$

$x-6 = -(2x+3)$

$x-6 = -2x-3$

$+2x \qquad +2x$

$3x-6 = -3$

$\quad +6 \quad +6$

$\frac{3x}{3} = \frac{3}{3}$

$x=1$

x=1 is the only solution

$x-6 = 2x+3$

$-x \qquad -x$

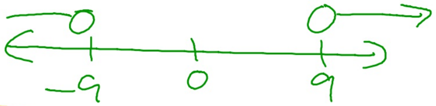
$-6 = x+3$

$-3 \qquad -3$

$\cancel{x=-9}$

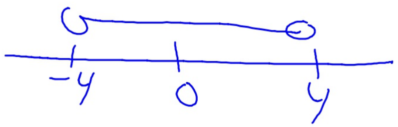
Solving Absolute Value Inequalities

What are all the x's that are more than 9 units from zero?



$x < -9$ or $x > 9$

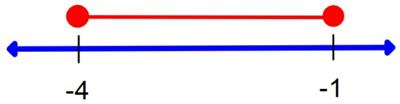
What are all the x's that are less than 4 units from zero?



$-4 < x < 4$

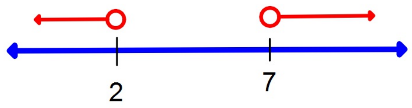
Write inequalities to model each graph.

1.



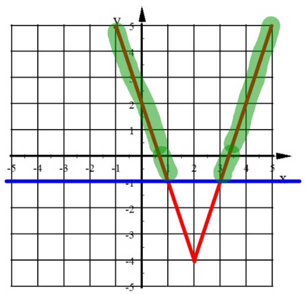
$-4 \leq x \leq -1$

2.



$x < 2$ or $x > 7$

Graph these equations together.



$y = 3|x - 2| - 4$ $\qquad$ $y = -1$

Use this graph to solve this inequality:

$3|x - 2| - 4 > -1$

another way to "read" this inequality is:
When is the V above the horizontal line?

The highlighted portion of the V is when
it is above ("greater than") the hizontal line.

Therefore, the solutions to this inequality are

x<1 or x>3

"to the left of 1 or to the right of 3"

Solve.

$$|x + 7| - 13 > 43$$

$\qquad +13 \qquad +13$

$$|x+7| > 56$$

DISTANCE from zero more than 56

$x+7 < -56$ or $x+7 > 56$

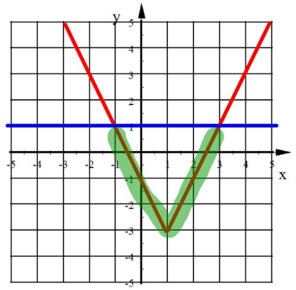
$-7 \quad -7 \qquad -7 \quad -7$

$x < -63$ or $x > 49$

Properties **Absolute Value Inequalities**

Let k represent a positive real number.

$|x| \geq k$ is equivalent to $x \leq -k$ or $x \geq k$.



Graph these equations together.

$y = 2|x + 1| - 3$ $y = 1$

Use this graph to solve this inequality:

$2|x + 1| - 3 \leq 1$

another way to "read" this inequality is:
When is the V below the horizontal line?

The highlighted portion of the V is when it is below ("less than") the hizontal line.

Therefore, the solutions to this inequality are

$-1 \leq x \leq 3$

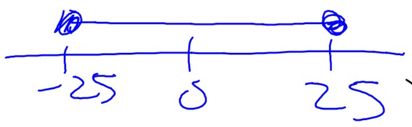
"all numbers between -1 and 3"

Solve. $|2x - 1| + 19 \leq 44$

$-19 \quad -19$

$|2x-1| \leq 25$

distance from zero is less than 25 units-----Between -25 & 25



$-25 \leq 2x-1 \leq 25$

$+1 \quad +1 \quad +1$

$\frac{-24}{2} \leq \frac{2x}{2} \leq \frac{26}{2}$

$\boxed{-12 \leq x \leq 13}$

Properties **Absolute Value Inequalities**

Let k represent a positive real number.

$|x| \leq k$ is equivalent to $-k \leq x \leq k$.