## Simple Interest vs. Compounded Interest

**Simple Interest**

Earning interest on the Principal (original amount) only.

**Compounded Interest**

The interest is added to the Principal each time. Therefore, you earn interest on the interest

## Simple Interest:

You invest $20,000 into an account that pays you a simple annual interest rate of 6%

1. How much interest will you earn each year?

| Principal | Yr # | Interest | Amount at end of year |
|---|---|---|---|
| $20,000 | 1 | (0.06)(20,000) = $1200 | 21,200 |
| $20,000 | 2 | (0.06)(20,000) = $1200 | 22,400 |
| $20,000 | 3 | (0.06)(20,000) = $1200 | 23,600 |
| $20,000 | 4 | (0.06)(20,000) = $1200 | 24,800 |
| $20,000 | 5 | (0.06)(20,000) = $1200 | 26,000 |

2. If you leave the money in this account and earn this interest every year how much will you have after 25 years?

$20000 + 25(1200) = 50,000$

2. If you leave the money in this account and earn this interest every year how much will you have after 25 years?

Simple Interest:

$$I = prt$$

- I: Interest earned
- p: Principal (initial investment)
- r: Interest rate as a decimal
- t: # yrs

$I = 20000(.06)(25)$

$= 30,000$

Total after 25 years = Principal + interest

= 20,000 + 30,000

= $50,000

## Compounding Interest:

You invest $20,000 into an account that pays you a 6% interest compounded annually.

| Beginning Principal | Yr # | Interest | Ending Amount | With Simple Interest: Amount at end of year |
|---|---|---|---|---|
| $20,000 | 1 | (0.06)(20,000) = $1200 | 21,200 | 21,200 |
| 21,200 | 2 | (0.06)(21200) = 1272 | 22,472 | 22,400 |
| 22,472 | 3 | (0.06)(22472) = 1348.32 | 23,820.32 | 23,600 |
| 23,820.32 | 4 | (0.06)(23820.32) = 1429.22 | 25,249.54 | 24,800 |
| 25,249.54 | 5 | (0.06)(25249.54) = 1514.97 | 26,764.51 | 26,000 |

How much will you have after 25 years? See next page

Compounding Interest Formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

**P** = principal amount (the initial amount you borrow or deposit)
**r** = annual rate of interest (as a decimal)
**t** = number of years the amount is deposited or borrowed for.
**A** = amount of money accumulated after n years, including interest.
**n** = number of times the interest is compounded per year

You invest $20,000 into an account that pays you a 6% interest compounded annually. How much will you have after 25 years?

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 20{,}000\left(1 + \frac{.06}{1}\right)^{1 \times 25}$$

$$= \$85{,}837.41$$

This turns out to be simple Exponential Growth! $y = a \cdot b^x$

You invest $20,000 in an account that pays 6% annual interest compounded MONTHLY. How much would you have after 25 years?

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 20{,}000\left(1 + \frac{.06}{12}\right)^{12 \cdot 25}$$

$$= \$89{,}299.40$$

The Number

**e**

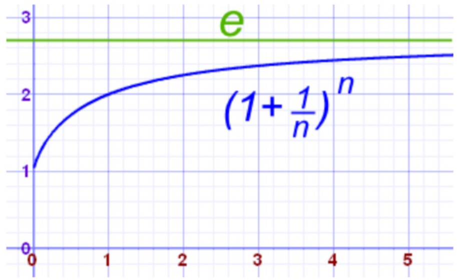
exploration

| Frequency of compounding | #times per year compound interest ($n$) | $1\left(1+\frac{1}{n}\right)^n$ | Dollar Value |
|---|---|---|---|
| Annually | $n = 1$ | $1\left(1+\frac{1}{1}\right)^1$ | 2.00 |
| Semiannually | $n = 2$ | $1\left(1+\frac{1}{2}\right)^2$ | 2.25 |
| quarterly | $n = 4$ | $1\left(1+\frac{1}{4}\right)^4$ | 2.441 |
| monthly | $n = 12$ | | 2.613 |
| weekly | $n = 52$ | | 2.693 |
| daily | $n = 365$ | | 2.715 |
| hourly | $n =$ 8760 | | 2.718 |
| every minute | $n =$ 525,600 | | 2.718 |
| every second | $n =$ 31,536,000 | | 2.718 |

As n increases it seems that this formula "leveling" out to a certain value.

This value is called e

the value of $(1 + 1/n)^n$ approaches **e** as n gets bigger and bigger:




| n | $(1 + 1/n)^n$ |
|---|---|
| 1 | 2.00000 |
| 2 | 2.25000 |
| 5 | 2.48832 |
| 10 | 2.59374 |
| 100 | 2.70481 |
| 1,000 | 2.71692 |
| 10,000 | 2.71815 |
| 100,000 | 2.71827 |

Where is e used?

Like π, e is most often found in formulas.

Equation of a Catenary: $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right)$

Catenary: A catenary is the shape that a cable assumes when it's supported at its ends and only acted on by its own weight.

Catenaris are used extensively in construction, especially for suspension bridges

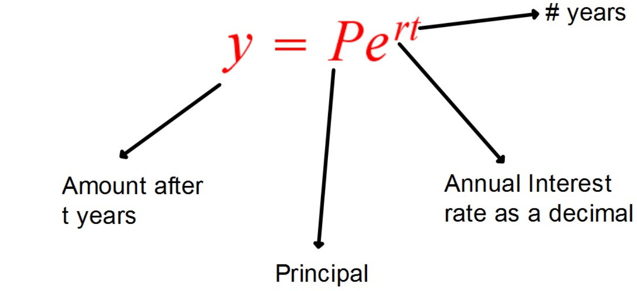
Mackinac Bridge

Another famous Catenary: The St. Louis Arch

The more often interest is calculated the more money you will earn.

What is more often than every second?

## Compounding Interest Continuously




You invest $20,000 in an account that pays 6% annual interest compounded CONTINUOUSLY. How much would you have after 25 years?

$$y = Pe^{rt} = 20{,}000\, e\wedge(.06 \cdot 25)$$

$89,633.78

You invest $20,000 in an account that pays 6% annual interest compounded CONTINUOUSLY. When would you have $150,000?

$$y = Pe^{rt}$$

$$\frac{150{,}000}{20{,}000} = \frac{20{,}000\, e^{.06t}}{20{,}000}$$

$$7.5 = e^{.06t}$$

$$\log_e 7.5 = .06t$$

$$\frac{\log 7.5}{\log e} = .06t$$

$$\frac{\frac{\log 7.5}{\log e}}{.06} = \frac{.06t}{.06}$$

$$t = 33.58 \text{ yrs}$$

$\log_e 7.5 = 0.06x$

$\log_e$ is called a Natural Logarithm

and is written as LN or Ln or ln

$\log_e 7.5 = 0.06x$

$\ln 7.5 = 0.06x$

Write in logaritmic form.

$5^x = 80$

$\log_5 80 = x$

$10^x = 137$

$\log 137 = x$

$e^x = 40$

$\log_e 40 = x \longrightarrow \ln 40 = x$

Write in exponential form:

$\log_x 12 = 3$

$x^3 = 12$

$\log 7 = x$

$10^x = 7$

$\ln x = 44$

$e^{44} = x$