Sec 14-1: Trigonometric Identities

An Identity is an equation where both sides are ALWAYS equal.

Example: $x + 2 + 7 + 2x = 3(x + 3)$

if you simplify both sides you get the following:

$$3x + 9 = 3x + 9$$

Tools available to you:

$$Tan\theta = \frac{Sin\theta}{Cos\theta} \qquad Cot\theta = \frac{1}{Tan\theta} = \frac{Cos\theta}{Sin\theta}$$

$$Csc\theta = \frac{1}{Sin\theta} \qquad Sec\theta = \frac{1}{Cos\theta}$$

You will be doing two things in this section.

- Simplifying trig expressions.
- Verifying a trig identity.

Simplify this trig expression.

$$\frac{Tan x}{Sec x} = \frac{\frac{Sin}{Cos}}{\frac{1}{Cos}}$$

$$= \frac{Sin}{\cancel{Cos}} \cdot \frac{\cancel{Cos}}{1} = \boxed{Sin x}$$

One technique that is used is to change everything into Sin and Cos then simplify.

Simplify this trig expression.

$$\frac{\text{Sec}x}{\text{Csc}x} = \frac{\frac{1}{\cos}}{\frac{1}{\sin}} = \frac{1}{\cos} \cdot \frac{\sin}{1} = \frac{\sin}{\cos}$$

$$= \boxed{\text{Tan}X}$$

Simplify this trig expression.

$(\text{Cos}x)(\text{Tan}x)(\text{Sin}x)$

$$\frac{\cancel{\cos}}{1} \cdot \frac{\sin}{\cancel{\cos}} \cdot \frac{\sin}{1} = (\sin X)^2 = \boxed{\sin^2 X}$$

Simplify this trig expression.

$\text{Sin}^2x \cdot \text{Cot}x \cdot \text{Csc}x$

$$\cancel{\sin^2} x \cdot \frac{\cos}{\cancel{\sin}} \cdot \frac{1}{\cancel{\sin}}$$

$\cos x$

Strategies for Simplifying Expressions

1) Change the expression into sines and cosines.
2) Look to use known formulas for purposes of substitution.
3) If there are fractions, gain a common denominator.
4) Use algebraic manipulations, like factoring, distributing, ...
5) If a strategy or substitution proves not to help, try something different.

The Pythagorean Identity

$x^2 + y^2 = 1$

This is the equation of the circle with a radius of 1 and center at (0,0)

Using the Unit Circle as the circle this equation becomes the Pythagorean Identity

$x^2 + y^2 = 1$

$\boxed{\sin^2 x + \cos^2 x = 1}$

$$\sin^2\theta + \cos^2\theta = 1$$

This can be rearranged to look like two "new" identities:

$\sin^2\theta + \cos^2\theta = 1$

$- \cos^2 x \qquad - \cos^2 x$

$\boxed{\sin^2\theta = 1 - \cos^2\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$- \sin^2 x \qquad - \sin^2 x$

$\boxed{\cos^2\theta = 1 - \sin^2\theta}$

Starting with $\sin^2 x + \cos^2 x = 1$

Two other Pythagorean Identities can be derived:

Divide both sides by $\sin^2 x$

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$\boxed{1 + \cot^2 x = \csc^2 x}$

Divide both sides by $\cos^2 x$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$\boxed{\tan^2 x + 1 = \sec^2 x}$

Trigonometric Tools:

Basic Identities:

$$\text{Tan}\theta = \frac{\text{Sin}\theta}{\text{Cos}\theta}$$

$$\text{Cot}\theta = \frac{1}{\text{Tan}\theta} = \frac{\text{Cos}\theta}{\text{Sin}\theta}$$

$$\text{Csc} = \frac{1}{\text{Sin}\theta}$$

$$\text{Sec} = \frac{1}{\text{Cos}\theta}$$

Pythagorean Identities:

$$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$$

$$\text{Sin}^2\theta = 1 - \text{Cos}^2\theta$$

$$\text{Cos}^2\theta = 1 - \text{Sin}^2\theta$$

$$\text{Tan}^2\theta + 1 = \text{Sec}^2\theta$$

$$1 + \text{Cot}^2\theta = \text{Csc}^2\theta$$

Simplify each trig expression:

You could also distribute the Cotθ. Since Cot and Tan are reciprocals their product is always just 1.

$$\text{Cot}\theta(\text{Tan}\theta + \text{Cot}\theta) = 1 + \text{cot}^2\theta = \boxed{\text{csc}^2\theta}$$

$$\frac{\cos}{\sin}\left(\frac{\sin}{\cos} + \frac{\cos}{\sin}\right)$$

You can always change functions in to Sin and Cos then simplify.

$$1 + \frac{\cos^2}{\sin^2} = 1 + \cot^2 = \boxed{\csc^2\theta}$$

$$\frac{\text{Sinx}}{1 - \text{Cos}^2\text{x}} = \frac{\sin x}{\sin^2 x} = \frac{1}{\sin x} = \boxed{\csc x}$$

$$\frac{\text{Tan}^2\text{x} + 1}{1 + \text{Cot}^2\text{x}} = \frac{\sec^2 x}{\csc^2 x} = \frac{\frac{1}{\cos^2}}{\frac{1}{\sin^2}} = \frac{1}{\cos^2} \cdot \frac{\sin^2}{1}$$

$$\frac{\sin^2}{\cos^2}$$

$$\text{Tan}^2 x$$